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CONSISTENCY AND EFFICIENT SOLUTION OF THE SYLVESTER EQUATION FOR $\star$-CONGRUENCE*

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Abstract. In this paper, the matrix equation $AX + X^\star B = C$ is considered, where the matrices A and B have sizes $m \times n$ and $n \times m$, respectively, the size of the unknown X is $n \times m$, and the operator $(\cdot)^\star$ denotes either the transpose or the conjugate transpose of a matrix. In the first part of the paper, necessary and sufficient conditions for the existence and uniqueness of solutions are reviewed. These conditions were obtained previously by Wimmer [H.K. Wimmer. Roth's theorems for matrix equations with symmetry constraints. *Linear Algebra Appl.*, 199:357–362, 1994.], by Byers and Kressner [R. Byers and D. Kressner. Structured condition numbers for invariant subspaces. *SIAM J. Matrix Anal. Appl.*, 28:326–347, 2006.], and by Kressner, Schröder and Watkins [D. Kressner, C. Schröder, and D.S. Watkins. Implicit QR algorithms for palindromic and even eigenvalue problems. *Numer. Algorithms*, 51:209–238, 2009.]. This review generalizes to fields of characteristic different from two the existence condition that Wimmer originally proved for the complex field. In the second part, an algorithm is developed, in the real or complex square case $m = n$, to solve the equation in $O(n^3)$ flops when the solution is unique. This algorithm is based on the generalized Schur decomposition of the matrix pencil $A - \lambda B^\star$. The equation $AX + X^\star B = C$ is connected with palindromic eigenvalue problems and, as a consequence, the square complex case has attracted recently the attention of several authors.

Key words. Generalized Schur decomposition, Matrix equations, Sylvester equation, Palindromic eigenvalue problems, Congruence of matrices.

AMS subject classifications. 65F05, 65F15, 15A24.

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