

STRUCTURE OF NILPOTENT MATRICES OVER FIELDS*

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Abstract. A zero-nonzero pattern \mathcal{A} is said to be potentially nilpotent over a field \mathbb{F} if there exists a nilpotent matrix with entries in \mathbb{F} having zero-nonzero pattern \mathcal{A} . We explore the construction of potentially nilpotent patterns over a field. We present classes of patterns which are potentially nilpotent over a field \mathbb{F} if and only if the field \mathbb{F} contains certain roots of unity. We then introduce some sparse patterns of order $n \geq 4$ which are spectrally arbitrary over \mathbb{C} but not over \mathbb{R} . We also identify all irreducible patterns of order four which are potentially nilpotent over \mathbb{R} or \mathbb{C} .

Key words. Nonzero pattern, Spectrum, Potentially nilpotent, Spectrally arbitrary, Nilpotent-Jacobian method.

AMS subject classifications. 15A18, 05C05, 05C50, 15B35.

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