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NORM PRESERVERS OF JORDAN PRODUCTS*

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Abstract. Norm preserver maps of Jordan product on the algebra M_n of $n \times n$ complex matrices are studied, with respect to various norms. A description of such surjective maps with respect to the Frobenius norm is obtained: Up to a suitable scaling and unitary similarity, they are given by one of the four standard maps (identity, transposition, complex conjugation, and conjugate transposition) on M_n, except for a set of normal matrices; on the exceptional set they are given by another standard map. For many other norms, it is proved that, after a suitable reduction, norm preserver maps of Jordan product transform every normal matrix to its scalar multiple, or to a scalar multiple of its conjugate transpose.

Key words. Jordan product, Matrix norm, Nonlinear preservers.

AMS subject classifications. 15A60, 15A86, 15A30.

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