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SHAPES AND COMPUTER GENERATION OF NUMERICAL RANGES OF KREIN SPACE OPERATORS*

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Dedicated to Hans Schneider on the occasion of his seventieth birthday.

Abstract. Let $(\mathcal{K}, \langle\cdot,\cdot\rangle)$ be a Hilbert space equipped with an indefinite inner product $[\cdot,\cdot]$. Then $(\mathcal{K}, [\cdot,\cdot])$ is a (complex) Krein space. One can define the Krein space numerical range of an operator A acting on $\mathcal{K}$ as the collection of complex numbers of the form $[Av, v]$ with $v \in \mathcal{K}$ satisfying $[v, v] = 1$. In this paper, the shapes of Krein space numerical ranges of operators in the complex plane using the joint numerical range of self-adjoint operators on $(\mathcal{K}, \langle\cdot,\cdot\rangle)$ are studied. Krein space numerical ranges of operators acting on a two-dimensional space are fully described. A Matlab program is developed to generate the sets in the finite dimensional case.

Key words. Indefinite inner product, Krein space, numerical range

AMS subject classifications. 15A60, 15A63.

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