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ELA

ON THE RODMAN-SHALOM CONJECTURE REGARDING THE JORDAN FORM OF COMPLETIONS OF PARTIAL UPPER TRIANGULAR MATRICES*

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Dedicated to Hans Schneider on the occasion of his seventieth birthday.

Abstract. Rodman and Shalom [*Linear Algebra and its Applications*, 168:221–249, 1992] conjecture the following statement: Let A be a lower irreducible partial upper triangular $n \times n$ matrix over F such that $trace(A) = 0$. Let $n_1 \geq n_2 \geq \cdots \geq n_p \geq 1$ be a set of p positive integers such that $\sum_{i=1}^{p} n_i = n$. There exists a nilpotent completion A_c of A whose Jordan form consists of p blocks of sizes $n_i \times n_i$, $i = 1, 2, \ldots, p$ if and only if $r(A^k) \leq \sum_{i:n_i \geq k} (n_i - k)$, $k = 1, 2, \ldots, n_1$.

In this paper this conjecture is solved in two cases: when the minimal rank of A is 2, and for matrices of size 5×5.

Key words. Completion problems, Jordan structure, Minimal rank, Majorization.

AMS subject classifications. 15A18, 15A21

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