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SIGN-CONSISTENCY AND SOLVABILITY OF CONSTRAINED LINEAR SYSTEMS*

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Abstract. Sign-solvable linear systems were introduced in modelling economic and physical systems where only qualitative information is known. Often economic and physical constraints require the entries of a solution to be nonnegative. Yet, to date the assumption of nonnegativity has been omitted in the study of sign-solvable linear systems. In this paper, the notions of sign-consistency and sign-solvability of a constrained linear system $Ax = b,\ x \gneqq 0$, are introduced. These notions give rise to new classes of sign patterns. The structure and the complexity of the recognition problem for each of these classes are studied. A qualitative analog of Farkas' Lemma is proven, and it is used to establish necessary and sufficient conditions for the constrained linear system $Ax = b,\ x \gneqq 0$ to be sign-consistent. Also, necessary and sufficient conditions for the constrained linear system $Ax = b,\ x \gneqq 0$ to be sign-solvable are determined, and these are used to establish a polynomial-time recognition algorithm. It is worth noting that the recognition problem for (unconstrained) sign-solvable linear systems is known to be NP-complete.

Key words. Linear systems, Qualitative Matrix Theory, Sign-solvable.

AMS subject classifications. 15A06, 90C08

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