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ON A STRONG FORM OF A CONJECTURE OF BOYLE AND HANDELMAN *

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Abstract. Let $\rho_{r,m}(x,\lambda) := (x-\lambda)^r \sum_{i=0}^{m} \binom{r+i-1}{i} x^{m-i}\lambda^i$. In this paper it is shown that if $\lambda_1, \ldots, \lambda_n$ are complex numbers such that $\lambda_1 = \lambda_2 = \ldots = \lambda_r > 0$ and $0 \leq \sum_{i=1}^{n} \lambda_i^k \leq n\lambda_1^k$, for $1 \leq k \leq m := n-r$, then

$$\prod_{i=1}^{n} (\lambda - \lambda_i) \ \leq \ \rho_{r,m}(\lambda, \lambda_1), \quad \text{for all} \ \ \lambda \geq 6.75\lambda_1. \ \ (*)$$

Moreover, if $r \geq m$, then $(*)$ holds for all $\lambda \geq \lambda_1$, while if $r < m$, but r is close to m, and n is large, one can lower the constant of 6.75 in the inequality $(*)$. The inequality $(*)$ is inspired by, and related to, a conjecture of Boyle and Handelman on the nonzero spectrum of a nonnegative matrix.

Key words. Nonnegative matrices, M-matrices, Inverse eigenvalue problem.

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