Electronic Journal of Linear Algebra ISSN 1081-3810 A publication of the International Linear Algebra Society Volume 22, pp. 1070-1084, November 2011



## POSITIVE AND RE-POSITIVE SOLUTIONS TO SOME SYSTEMS OF ADJOINTABLE OPERATOR EQUATIONS OVER HILBERT $C^*$ – MODULES\*

GUANG JING SONG<sup> $\dagger$ </sup> AND QING WEN WANG<sup> $\ddagger$ </sup>

**Abstract.** A necessary and sufficient condition for the existence of the general common positive solution to equations

$$A_1X = C_1, \ XB_2 = C_2, \ A_3XA_3^* = C_3, \ A_4XA_4^* = C_4$$

for operators between Hilbert  $C^*$ -modules is established, and an expression for the common positive solution to the equations is derived when the solvability conditions are satisfied. As an application, a new necessary and sufficient condition for the system of adjointable operator equations

$$AX = C, XB = D$$

over Hilbert  $C^*$ -modules to have a common Re-positive solution is proved. Moreover, an expression of the general Re-positive solution is derived when the consistent conditions are met. The results of this paper extend some known results in the literature.

Key words. Hilbert  $C^*$ -module, Moore-Penrose inverse, Operator equation, Positive solution, Re-positive solution.

AMS subject classifications. 47A62, 47B15, 47B65, 15A09, 15A24.

<sup>\*</sup>Received by the editors on February 23, 2011. Accepted for publication on October 4, 2011. Handling Editor: Michael Tsatsomeros.

<sup>&</sup>lt;sup>†</sup>School of Mathematics and Information Sciences, Weifang University Weifang 261061, P.R. China (sgjshu@yahoo.com.cn). Supported by grants from Natural Science Foundation of Shan Dong Province (no. ZR2010AL107) and the Ph.D. Foundation of Weifang University (no. 2011BS12).

<sup>&</sup>lt;sup>‡</sup>Department of Mathematics, Shanghai University, Shanghai 200444, P.R. China (wqw858@yahoo.com.cn). Supported by grants from the National Natural Science Foundation of China (no. 11171205), the Natural Science Foundation of Shanghai (no. 11ZR1412500), and the Ph.D. Programs Foundation of Ministry of Education of China (no. 20093108110001).