The Bifurcation Set for the 1:4 Resonance Problem

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Abstract

We study the bifurcation set in (b, ϕ, α)-space of the equation $\dot{z} = e^{i\alpha} z + e^{i\phi} z\,|z|^2 + b\bar{z}^3$. This $\mathbb{Z}_4$-equivariant planar vector field is equivalent to the model equation that has been considered in the study of the 1:4 resonance problem.

We present a three-dimensional model of the bifurcation set that describes the known properties of the system in a condensed way, and, under certain assumptions for which there is strong numerical evidence, is topologically correct and complete. In this model, the bifurcation set consists of surfaces of codimension-one bifurcations that divide (b, ϕ, α)-space into fifteen regions of generic phase portraits. The model also offers further insight into the question of versality of the system. All bifurcation phenomena seem to unfold generically for $\phi \neq \pi/2,\ 3\pi/2$.