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ETNA
Kent State University
etna@mcs.kent.edu

A CHEBYSHEV-LIKE SEMIITERATION FOR INCONSISTENT LINEAR SYSTEMS *

MARTIN HANKE† AND MARLIS HOCHBRUCK‡

Dedicated to Wilhelm Niethammer on the occasion of his 60th birthday.

Abstract. Semiiterative methods are known as a powerful tool for the iterative solution of nonsingular linear systems of equations. For singular but consistent linear systems with coefficient matrix of index one, one can still apply the methods designed for the nonsingular case. However, if the system is inconsistent, the approximations usually fail to converge. Nevertheless, it is still possible to modify classical methods like the Chebyshev semiiterative method in order to fulfill the additional convergence requirements caused by the inconsistency. These modifications may suffer from instabilities since they are based on the computation of the diverging Chebyshev iterates. In this paper we develop an alternative algorithm which allows to construct more stable approximations. This algorithm can be efficiently implemented with short recurrences. There are several reasons indicating that the new algorithm is the most natural generalization of the Chebyshev semiiteration to inconsistent linear systems.

Key words. Semiiterative methods, singular systems, Zolotarev problem, orthogonal polynomials.

AMS subject classifications. 65F10, 65F20.

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† Institut für Praktische Mathematik, Universität Karlsruhe, D–76128 Karlsruhe, Germany, (email: `hanke@ipmsun1.mathematik.uni-karlsruhe.de`).

‡ Institut für Angewandte Mathematik und Statistik, Universität Würzburg, Am Hubland, D–97074 Würzburg, Germany, (email: `marlis@mathematik.uni-wuerzburg.de`).