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GENERAL HIGHLY ACCURATE ALGEBRAIC COARSENING*

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Abstract. General purely algebraic approaches for repeated coarsening of deterministic or statistical field equations are presented, including a universal way to gauge and control the quality of the coarse-level set of variables, and generic procedures for deriving the coarse-level set of equations. They apply to the equations arising from variational as well as non-variational discretizations of general, elliptic as well as non-elliptic, partial differential systems, on structured or unstructured grids. They apply to many types of disordered systems, such as those arising in composite materials, inhomogeneous ground flows, "twisted geometry" discretizations and Dirac equations in disordered gauge fields, and also to non-PDE systems. The coarsening can be inexpensive with low accuracy, as needed for multigrid solvers, or more expensive and highly accurate, as needed for other applications (e.g., once-for-all derivation of macroscopic equations). Extensions to non-local and highly indefinite (wave) operators are briefly discussed. The paper re-examines various aspects of algebraic multigrid (AMG) solvers, suggesting some new approaches for relaxation, for interpolation, and for convergence acceleration by recombining iterants. An application to the highly-disordered Dirac equations is briefly reviewed.

Key words. multiscale algorithms, multigrid, algebraic multigrid, AMG, nonlinear AMG, unstructured grids, coarsening, distributive coarsening, homogenization, compatible relaxation, Dirac equations.

AMS subject classifications. 35A40, 65F10, 65K10, 65M55, 65N22, 65N55, 65Y05, 76M20.

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