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CONTINUOUS Θ-METHODS FOR THE STOCHASTIC PANTOGRAPH EQUATION*

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Abstract. We consider a stochastic version of the pantograph equation:

$$dX(t) = \{aX(t) + bX(qt)\}\,dt + \{\sigma_1 + \sigma_2 X(t) + \sigma_3 X(qt)\}\,dW(t),$$
$$X(0) = X_0,$$

for $t \in [0, T]$, a given Wiener process W and $0 < q < 1$. This is an example of an Itô stochastic delay differential equation with unbounded memory. We give the necessary analytical theory for existence and uniqueness of a strong solution of the above equation, and of strong approximations to the solution obtained by a continuous extension of the Θ-Euler scheme ($\Theta \in [0, 1]$). We establish $\mathcal{O}(\sqrt{h})$ mean-square convergence of approximations obtained using a bounded mesh of uniform step h, rising in the case of additive noise to $\mathcal{O}(h)$. Illustrative numerical examples are provided.

Key words. stochastic delay differential equation, continuous Θ-method, mean-square convergence.

AMS subject classifications. 65C30, 65Q05.

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