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Contents

1 Numerical experiments with algebraic multilevel preconditioner. *Gérard Meurant.*

Abstract.
This paper numerically compares different algebraic multilevel preconditioners to solve symmetric positive definite linear systems with the preconditioned conjugate gradient algorithm on a set of examples arising mainly from discretization of second order partial differential equations. We compare several different smoothers, influence matrices and interpolation schemes.

Key Words.
multilevel preconditioner, conjugate gradient.

AMS(MOS) Subject Classifications.
65F10, 65F50.

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vol.12.2001/pp1-65.dir/pp1-65.ps;
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Forward References.

66 Numerical condition of polynomials in different forms. *Hong Zhang.*

Abstract.
The zeros of high-degree polynomials are notoriously sensitive to changes in the coefficients, causing problems for available zero-finding software. In this paper, we study how this sensitivity depends on the polynomial representation. We first extend the algebraic characterization of polynomial pseudozero sets from the power basis to general bases. We show that for a polynomial, the numerical conditions of its values and zeros are closely related and can be visualized simultaneously by its pseudozero sets. Comparing the pseudozero sets on a set of testing polynomials in the power, Taylor, Chebyshev, and Bernstein bases reveals that appropriate representation of polynomials gives rise to locally well-conditioned zeros, which then leads to an Iterative Refinement Algorithm that combines symbolic formulation with numeric processing to reduce computational errors of polynomial zeros located in the region of interest.

Key Words.
polynomial equation, polynomial basis, pseudozero set, iterative refinement algorithm.

AMS(MOS) Subject Classifications.
65F35.

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Forward References.

88 On Parallel two-stage methods for Hermitian positive definite matrices with applications to preconditioning. *M. Jesús Castel, Violeta Migallón and José Penadés.*

Abstract.
Parallel two-stage iterative methods for the solution of linear systems are analyzed. Convergence properties of both block and multisplitting two-stage methods are investigated either when the number of inner iterations becomes sufficiently large or when the matrix of the linear system is Hermitian positive definite. Comparison theorems for the parallel two-stage methods, based on the number of inner iterations performed, are given. Computational results of these methods on two parallel computing systems are included to illustrate the theoretical results. Also, the use of these methods as preconditioners is studied from the experimental point of view.

Key Words.
linear systems, two-stage methods, block methods, multisplitting methods, Hermitian matrix, positive definite matrix, preconditioners, parallel algorithms, monotonicity, distributed memory.

AMS(MOS) Subject Classifications.
65F10, 65F15.

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vol.12.2001/pp88-112.dir/pp88-112.ps;
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Forward References.

113 Geršgorin-type eigenvalue inclusion theorems and their dharpness. *Richard S. Varga.*

Abstract.

Here, we investigate the relationships between $\mathcal{G}(A)$, the union of Geršgorin disks, $\mathcal{K}(A)$, the union of Brauer ovals of Cassini, and $\mathcal{B}(A)$, the union of Brualdi lemniscate sets, for eigenvalue inclusions of an $n \times n$ complex matrix A. If $\sigma(A)$ denotes the spectrum of A, we show here that

$$\sigma(A) \subseteq \mathcal{B}(A) \subseteq \mathcal{K}(A) \subseteq \mathcal{G}(A)$$

is valid for any weakly irreducible $n \times n$ complex matrix A with $n \geq 2$. Further, it is evident that $\mathcal{B}(A)$ can contain the spectra of related $n \times n$ matrices. We show here that the spectra of these related matrices can fill out $\mathcal{B}(A)$. Finally, if $\mathcal{G}^{\mathcal{R}}(A)$ denotes the minimal Geršgorin set for A, we show that

$$\mathcal{G}^{\mathcal{R}}(A) \subseteq \mathcal{B}(A).$$

Key Words.
Geršgorin disks, Brauer ovals of Cassini, Brualdi lemniscate sets, minimal Geršgorin sets.

AMS(MOS) Subject Classifications.
15A18.

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vol.12.2001/pp113-133.dir/pp113-133.ps;
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Forward References.

134 Some nonstandard finite element estimates with applications to 3D Poisson and Signorini problems . *Faker Ben Belgacem and Susanne C. Brenner.*

Abstract.
In this paper we establish several nonstandard finite element estimates involving fractional order Sobolev spaces, with applications to bubble stabilized mixed methods for the three-dimensional Poisson and Signorini problems.

Key Words.
fractional order Sobolev spaces, Poisson, Signorini, mixed method, bubble finite element stabilization.

AMS(MOS) Subject Classifications.
65N30, 35J85.

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vol.12.2001/pp134-148.dir/pp134-148.ps;
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Forward References.

149 Piecewise linear wavelet collocation, approximation of the boundary manifold, and quadrature. *S. Ehrich and A. Rathsfeld.*

Abstract.

In this paper we consider a piecewise linear wavelet collocation method for the solution of boundary integral equations of order $\mathbf{r} = 0, -1$ over a closed and smooth boundary manifold. The trial space is the space of all continuous and piecewise linear functions defined over a uniform triangular grid and the collocation points are the grid points. For the wavelet basis in the trial space we choose the three-point hierarchical basis together with a slight modification near the boundary points of the global patches of parametrization. We choose three, four, and six term linear combinations of Dirac delta functionals as wavelet basis in the space of test functionals. The usual compression results apply, i.e., for N degrees of freedom, the fully populated stiffness matrix of N^2 entries can be approximated by a sparse matrix with no more than $\mathcal{O}(N[\log N]^2)$ nonzero entries. The topic of the present paper, however, is to show that the parametrization can be approximated by low order piecewise polynomial interpolation and that the integrals in the stiffness matrix can be computed by quadrature, where the quadrature rules are combinations of product integration applied to non analytic factors of the integrand and of high order Gauß rules applied to the analytic parts. The whole algorithm for the assembling of the matrix requires no more than $\mathcal{O}(N[\log N]^4)$ arithmetic operations, and the error of the collocation approximation, including the compression, the approximate parametrization, and the quadratures, is less than $\mathcal{O}(N^{-1}[\log N]^2)$. Note that, in contrast to well-known algorithms by v.Petersdorff, Schwab, and Schneider, only a finite degree of smoothness is required.

Key Words.
boundary integral equation of order 0 and -1, piecewise linear collocation, wavelet algorithm, approximation of parametrization, quadrature.

AMS(MOS) Subject Classifications.
45L10, 65D32, 65R20, 65N38.

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vol.12.2001/pp149-192.dir/pp149-192.ps;
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Forward References.

193 Multi-symplectic Fourier pseudospectral method for the nonlinear Schrödinger equation. *Jing-Bo Chen and Meng-Zhao Qin.*

Abstract.

Bridges and Reich suggested the idea of multi-symplectic spectral discretization on Fourier space [4]. Based on their theory, we investigate the multi-symplectic Fourier pseudospectral discretization of the nonlinear Schrödinger equation (NLS) on real space. We show that the multi-symplectic semi-discretization of the nonlinear Schrödinger equation with periodic boundary conditions has N (the number of the nodes) semi-discrete multi-symplectic conservation laws. The symplectic discretization in time of the semi-discretization leads to N full-discrete multi-symplectic conservation laws. We also prove a result relating to the spectral differentiation matrix. Numerical experiments are included to demonstrate the remarkable local conservation properties of multi-symplectic spectral discretizations.

Key Words.
Multi-symplectic, Fourier pseudospectral method, nonlinear Schrödinger equation.

AMS(MOS) Subject Classifications.
65M99.

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vol.12.2001/pp193-204.dir/pp193-204.pdf;

Forward References.

205 Chebyshev approximation via polynomial mappings and the convergence behaviour of Krylov subspace methods. *Bernd Fischer and Franz Peherstorfer.*

Abstract.

Let φ_m be a polynomial satisfying some mild conditions. Given a set $R \subset \mathbb{C}$, a continuous function f on R and its best approximation p_{n-1}^* from Π_{n-1} with respect to the maximum norm, we show that $p_{n-1}^* \circ \varphi_m$ is a best approximation to $f \circ \varphi_m$ on the inverse polynomial image S of R, i.e. $\varphi_m(S) = R$, where the extremal signature is given explicitly. A similar result is presented for constrained Chebyshev polynomial approximation. Finally, we apply the obtained results to the computation of the convergence rate of Krylov subspace methods when applied to a preconditioned linear system. We investigate pairs of preconditioners where the eigenvalues are contained in sets S and R, respectively, which are related by $\varphi_m(S) = R$.

Key Words.
Chebyshev polynomial, optimal polynomial, extremal signature, Krylov subspace method, convergence rate.

AMS(MOS) Subject Classifications.
41A10, 30E10, 65F10.

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Forward References.

216 Retooling the method of block conjugate gradients. *A. A. Dubrulle.*

Abstract.
Block implementations of the conjugate-gradients method for the solution of a linear system must deal with linear dependences that may appear in the descent or residual blocks in the course of the iteration. New algorithms presented here avoid rank estimation and deflation through the use of changes of bases and algorithmic reformulations that eliminate rank near defects. The transformations include a robust process of nonunitary orthogonalisation in the metric of a symmetric positive-definite matrix.

Key Words.
linear equations, conjugate gradients, orthogonalisation.

AMS(MOS) Subject Classifications.
65F10, 65F25.

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vol.12.2001/pp216-233.dir/pp216-233.ps;
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Forward References.

234 Error Analysis of QR Algorithms for Computing Lyapunov Exponents. *Edward J. McDonald and Desmond J. Higham.*

Abstract.
Lyapunov exponents give valuable information about long term dynamics. The discrete and continuous QR algorithms are widely used numerical techniques for computing approximate Lyapunov exponents, although they are not yet supported by a general error analysis. Here, a rigorous convergence theory is developed for both the discrete and continuous QR algorithm applied to a constant coefficient linear system with real distinct eigenvalues. For the discrete QR algorithm, the problem essentially reduces to one of linear algebra for which the timestepping and linear algebra

errors uncouple and precise convergence rates are obtained. For the continuous QR algorithm, the stability, rather than the local accuracy, of the timestepping algorithm is relevant, and hence the overall convergence rate is independent of the stepsize. In this case it is vital to use a timestepping method that preserves orthogonality in the system. We give numerical results to illustrate the analysis. Further numerical experiments and a heuristic argument suggest that the convergence properties carry through to the case of complex conjugate eigenvalue pairs.

Key Words.
dynamics, eigenvalues, orthogonal iteration, timestepping.

AMS(MOS) Subject Classifications.
65L05, 65F15.

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Forward References.