Electronic Transactions on Numerical Analysis.
Volume 12, pp. 193-204, 2001.

ISSN 1068-9613.

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MULTI-SYMPLECTIC FOURIER PSEUDOSPECTRAL METHOD FOR THE NONLINEAR SCHRÖDINGER EQUATION *

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Abstract. Bridges and Reich suggested the idea of multi-symplectic spectral discretization on Fourier space [4]. Based on their theory, we investigate the multi-symplectic Fourier pseudospectral discretization of the nonlinear Schrödinger equation (NLS) on real space. We show that the multi-symplectic semi-discretization of the nonlinear Schrödinger equation with periodic boundary conditions has N (the number of the nodes) semi-discrete multi-symplectic conservation laws. The symplectic discretization in time of the semi-discretization leads to N full-discrete multi-symplectic conservation laws. We also prove a result relating to the spectral differentiation matrix. Numerical experiments are included to demonstrate the remarkable local conservation properties of multi-symplectic spectral discretizations.

Key words. Multi-symplectic, Fourier pseudospectral method, nonlinear Schrödinger equation.

AMS subject classifications. 65M99.

*Received April 26, 2000. Accepted for publication June 13, 2001. Recommended by V. Druskin.

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