Electronic Transactions on Numerical Analysis
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Contents

1 A uniformly accurate finite volume discretization for a convection-diffusion problem.
Dirk Wollstein, Torsten Linss and Hans-Görg Roos.

Abstract.
A singularly perturbed convection-diffusion problem is considered. The problem is discretized using an inverse-monotone finite volume method on Shishkin meshes. We establish first-order convergence in a global energy norm and a mesh-dependent discrete energy norm, no matter how small the perturbation parameter. Numerical experiments support the theoretical results.

Key Words.
convection-diffusion problems, finite volume methods, singular perturbation, Shishkin mesh.

AMS(MOS) Subject Classifications.
65N30.

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vol.13.2002/pp1-11.dir/pp1-11.ps;
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Forward References.

12 The asymptotic distribution of general interpolation arrays for exponential weights.
S. B. Damelin.

Abstract.
We study the asymptotic distribution of general interpolation arrays for a large class of even exponential weights on the line and $(-1, 1)$. Our proofs rely on deep properties of logarithmic potentials. We conclude with some open problems.

Key Words.
asymptotic distribution, Freud weight, Erds weight, exponential weight, interpolation, Lebesgue constant, logarithmic potential, Pollaczek weight, sup norm, weighted approximation.

AMS(MOS) Subject Classifications.
42C15, 42C05, 65D05.

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vol.13.2002/pp12-21.dir/pp12-21.ps;
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Forward References.

22 Image restoration through subimages and confidence images. *James G. Nagy and Dianne P. O'Leary.*

Abstract.
Some very effective but expensive image reconstruction algorithms cannot be applied to large images because of their cost. In this work, we first show how to apply such algorithms to subimages, giving improved reconstruction of regions of interest. Our second contribution is to construct confidence intervals for pixel values, by generalizing a theorem of O'Leary and Rust to allow both upper and lower bounds on variables. All current algorithms for image deblurring or deconvolution output an image. This provides an estimated value for each pixel in the image. What is lacking is an estimate of the statistical confidence that we can have in those pixel values or in the features they form in the image. There are two obstacles in determining confidence intervals for pixel values: first, the process is computationally quite intensive, and second, there has been no proposal for providing the results in a visually useful way. In this work we overcome the first of those limitations and develop an algorithm called Twinkle to overcome the second. We demonstrate the usefulness of these techniques on astronomical and motion-blurred images.

Key Words.
image restoration, regularization, confidence intervals, confidence images, motion blur, conjugate gradient method.

AMS(MOS) Subject Classifications.
65F10, 65F20, 65F30.

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vol.13.2002/pp22-37.dir/pp22-37.ps;
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Forward References.

38 Perturbation of parallel asynchronous linear iterations by floating point errors. *Pierre Spiteri, Jean-Claude Miellou and Didier El Baz.*

Abstract.
This paper deals with parallel asynchronous linear iterations perturbed by errors in floating point arithmetic. An original result is presented which permits one to localize the limits of perturbed parallel asynchronous linear iterations. The result is established by using the approximate contraction concept. Simple examples are studied.

Key Words.
approximate contraction, parallel algorithms, asynchronous iterations.

AMS(MOS) Subject Classifications.
65F10, 65G05, 65Y05, 68Q22, 68Q10.

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vol.13.2002/pp38-55.dir/pp38-55.ps;
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Forward References.

56 On error estimation in the conjugate gradient method and why it works in finite precision computations. *Zdeněk Strakoš and Petr Tichý.*

Abstract.
In their paper published in 1952, Hestenes and Stiefel considered the conjugate gradient (CG) method an iterative method which terminates in at most n steps if no rounding errors are encountered [p. 410]. , They also proved identities for the A-norm and the Euclidean norm of the error which could justify the stopping criteria [Theorems 6:1 and 6:3, p. 416]. , The idea of estimating errors in iterative methods, and in the CG method in particular, was independently (of these results) promoted by Golub; the problem was linked to Gauss quadrature and to its modifications [7], [8]. A comprehensive summary of this approach was given in [15], [16]. During the last decade several papers developed error bounds algebraically without using Gauss quadrature. However, we have not found any reference to the corresponding results in [24]. All the existing bounds assume exact arithmetic. Still they seem to be in a striking agreement with finite precision numerical experiments, though in finite precision computations they estimate quantities which can be orders of magnitude different from their exact precision counterparts! For the lower bounds obtained from Gauss quadrature formulas this nontrivial phenomenon was explained, with some limitations, in [17].
In our paper we show that the lower bound for the A-norm of the error based on Gauss quadrature ([15], [17], [16]) is mathematically equivalent to the original formula of Hestenes and Stiefel [24]. We will compare existing bounds and we will demonstrate necessity of a proper rounding error analysis: we present an example of the well-known bound which can fail in finite precision arithmetic. We will analyse the simplest bound based on [24, Theorem 6:1], and prove that it is numerically stable. Though we concentrate mostly on the lower bound for the A-norm of the error, we describe also an estimate for the Euclidean norm of the error based on [24, Theorem 6:3]. Our results are illustrated by numerical experiments.

Key Words.
conjugate gradient method, Gauss quadrature, evaluation of convergence, error bounds, finite precision arithmetic, rounding errors, loss of orthogonality.

AMS(MOS) Subject Classifications.
15A06, 65F10, 65F25, 65G50.

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vol.13.2002/pp56-80.dir/pp56-80.ps;
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81 Multigrid preconditioning and Toeplitz matrices. *Thomas Huckle and Jochen Staudacher.*

Abstract.

In this paper we discuss multigrid methods for symmetric Toeplitz matrices. Then the restriction and prolongation operators can be seen as projected Toeplitz matrices. Because of the intimate connection between such matrices and trigonometric series we can express the multigrid algorithm in terms of the underlying functions with special zeros. This shows how to choose the prolongation/restriction operator in order to get fast convergence. We start by considering Toeplitz matrices with generating functions having a single zero of finite order in , and we extend previous results on multigrid for Toeplitz matrices, in particular, by introducing a natural coarse grid operator. Afterwards we carry over our reasoning to cases with more than one zero and, we study how the previous cases relate to Toeplitz systems resulting from the discretization of Fredholm integral equations of the first kind which arise in image processing. Next, we take a brief look at Block Toeplitz systems with Toeplitz Blocks. We show how the one-dimensional techniques can be carried over easily for positive definite problems with a single zero in and we also present a multigrid algorithm for linear systems arising from practical image deblurring problems. Finally, we give a new characterization of the well-known difficulties encountered in the indefinite case.

Key Words.

multigrid methods, iterative methods, preconditioning, Toeplitz matrices, Fredholm integral equations, image deblurring.

AMS(MOS) Subject Classifications.

65N55, 65F10, 65F22, 65F35, 65R20.

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Forward References.

106 Polynomial eigenvalue problems with Hamiltonian structure. *Volker Mehrmann and David Watkins.*

Abstract.
We discuss the numerical solution of eigenvalue problems for matrix polynomials, where the coefficient matrices are alternating symmetric and skew symmetric or Hamiltonian and skew Hamiltonian. We discuss several applications that lead to such structures. Matrix polynomials of this type have a symmetry in the spectrum that is the same as that of Hamiltonian matrices or skew-Hamiltonian/Hamiltonian pencils. The numerical methods that we derive are designed to preserve this eigenvalue symmetry. We also discuss linearization techniques that transform the polynomial into a skew-Hamiltonian/Hamiltonian linear eigenvalue problem with a specific substructure. For this linear eigenvalue problem we discuss special factorizations that are useful in shift-and-invert Krylov subspace methods for the solution of the eigenvalue problem. We present a numerical example that demonstrates the effectiveness of our approach.

Key Words.
matrix polynomial, Hamiltonian matrix, skew-Hamiltonian matrix, skew-Hamiltonian/Hamiltonian pencil, matrix factorizations.

AMS(MOS) Subject Classifications.

65F15, 15A18, 15A22.

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vol.13.2002/pp106-118.dir/pp106-118.ps;
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Forward References.

119 The interplay between classical analysis and (numerical) linear algebra — a tribute to Gene H. Golub. *Walter Gautschi.*

Abstract.

Much of the work of Golub and his collaborators uses techniques of linear algebra to deal with problems in analysis, or employs tools from analysis to solve problems arising in linear algebra. Instances are described of such interdisciplinary work, taken from quadrature theory, orthogonal polynomials, and least squares problems on the one hand, and error analysis for linear algebraic systems, element-wise bounds for the inverse of matrices, and eigenvalue estimates on the other hand.

Key Words.

Gauss-type quadratures, eigenvalue/vector characterizations, orthogonal polynomials, modification algorithms, polynomials orthogonal on several intervals, least squares problem, Lanczos algorithm, bounds for matrix functionals, iterative methods.

AMS(MOS) Subject Classifications.

65D32, 33C45, 65D10, 15A45, 65F10.

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Forward References.