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Contents

Abstract.
Recently, we gave a theoretical explanation for superlinear convergence behavior observed while solving large symmetric systems of equations using the Conjugate Gradient method. Roughly speaking, one may observe superlinear convergence while solving a sequence of (symmetric positive definite) linear systems if the asymptotic eigenvalue distribution of the sequence of the corresponding matrices of coefficients is far from an equilibrium distribution. However, it is well known that the convergence of the Conjugate Gradient or other Krylov subspace methods does not only depend on the spectrum but also on the right-hand side of the underlying system and the starting vector. In this paper we present a family of examples based on the discretization via finite differences of the one dimensional Poisson problem where the asymptotic distribution equals an equilibrium distribution but one may as well observe superlinear convergence according to the particular choice of the right-hand sides. Our findings are related to some recent results concerning asymptotics of discrete orthogonal polynomials. An important tool in our investigations is a constrained energy problem in logarithmic potential theory, where an additional external field is used being related to our particular right-hand sides.

Key Words.
Superlinear convergence, Conjugate gradients, Krylov subspace methods, Logarithmic potential theory.

AMS(MOS) Subject Classifications.
65F10, 65E05, 31A99, 41A10.

Files.
vol.14.2002/pp1-19.dir/pp1-19.ps;
vol.14.2002/pp1-19.dir/pp1-19.pdf;

Forward References.

Abstract.
The L-curve is often applied to determine a suitable value of the regularization parameter when solving ill-conditioned linear systems of equations with a right-hand side contaminated by errors of unknown norm. The location of the vertex of the L-curve typically yields a suitable value of the regularization parameter. However, the computation of the L-curve and of its curvature is quite costly for large problems; the determination of a point on the L-curve requires that both the norm of the

regularized approximate solution and the norm of the corresponding residual vector be available. Recently, the L-ribbon, which contains the L-curve in its interior, has been shown to be suitable for the determination of the regularization parameter for large-scale problems. In this paper we describe how techniques similar to those employed for the computation of the L-ribbon can be used to compute a "curvature-ribbon," which contains the graph of the curvature of the L-curve. Both curvature- and L-ribbon can be computed fairly inexpensively by partial Lanczos bidiagonalization of the matrix of the given linear system of equations. A suitable value of the regularization parameter is then determined from these ribbons, and we show that an associated approximate solution of the linear system can be computed with little additional work.

Key Words.
Ill-posed problem, regularization, L-curve, Gauss quadrature.

AMS(MOS) Subject Classifications.
Ill-posed problem, regularization, L-curve, Gauss quadrature.

Files.
vol.14.2002/pp20-35.dir/pp20-35.ps;
vol.14.2002/pp20-35.dir/pp20-35.pdf;

Forward References.

Abstract.
Harmonic Hilbert spaces were introduced as an extension of periodic Hilbert spaces introduced by Babuka to the non-periodic case . In this paper we will investigate approximation by minimum norm interpolation in harmonic Hilbert spaces.

Key Words.
minimum norm interpolation, harmonic Hilbert spaces, remainders.

AMS(MOS) Subject Classifications.
41A05, 41A25, 41A65, 65D05, 65T99.

Files.
vol.14.2002/pp36-44.dir/pp36-44.ps;
vol.14.2002/pp36-44.dir/pp36-44.pdf;

Forward References.

Abstract.
This article presents a new algorithm for nonharmonic signal analysis using Dirichlet series on a convex polygon as a generalization of Fourier series. Here denotes a quasipolynomial whose set of zeros generates a Riesz basis of the Smirnov space. The algorithm is based on a simple form of and on numerical properties of the dual basis of .

Key Words.
nonharmonic Fourier series, Dirichlet series, signal analysis, time series analysis.

AMS(MOS) Subject Classifications.
42C15, 30B50, 37M10.

Files.
vol.14.2002/pp45-55.dir/pp45-55.ps;
vol.14.2002/pp45-55.dir/pp45-55.pdf;

Forward References.

56 On Multiscale denoising of spherical functions: basic theory and numerical aspects. *W. Freeden and T. Maier.*

Abstract.
The basic concepts of selective multiscale reconstruction of functions on the sphere from error–affected data is outlined for scalar functions. The selective reconstruction mechanism is based on the premise that multiscale approximation can be well–represented in terms of only a relatively small number of expansion coefficients at various resolution levels. A new proof, including non-bandlimited kernel functions, of the pyramid scheme is presented to efficiently remove the noise at different scales using a priori statistical information, i.e. knowledge of the covariance function.

Key Words.
spherical wavelet theory, scalar multiscale approximation, pyramid scheme, spectral and multiscale variance-covariance model, hard and soft thresholding.

AMS(MOS) Subject Classifications.
33C55, 42C40, 62-07, 65T60, 86A25.

Files.
vol.14.2002/pp56-78.dir/pp56-78.ps;
vol.14.2002/pp56-78.dir/pp56-78.pdf;

Forward References.

79 A polynomial collocation method for cauchy singular integral equations over the interval. *P. Junghanns and A. Rathsfeld.*

Abstract.
In this paper we consider a polynomial collocation method for the numerical solution of a singular integral equation over the interval. More precisely, the operator of our integral equation is supposed to be of the form with the Cauchy integral operator, with piecewise continuous coefficients and with a regular integral operator and with a Jacobi weight . To the equation we apply a collocation method, where the collocation points are the Chebyshev nodes of the second kind and where the trial space is the space of polynomials multiplied by another Jacobi weight. For the stability and convergence of this collocation in weighted spaces, we derive necessary and sufficient conditions.

Key Words.
Cauchy singular integral equation; polynomial collocation method; stability.

AMS(MOS) Subject Classifications.
45L10, 65R20, 65N38.

Files.
vol.14.2002/pp79-126.dir/pp79-126.ps;
vol.14.2002/pp79-126.dir/pp79-126.pdf;

Forward References.

127 Recent trends on analytic properties of matrix orthonormal polynomials. *F. Marcellán and H. O. Yakhlef.*

Abstract.
In this paper we give an overview of recent results on analytic properties of matrix orthonormal polynomials. We focus our attention on the distribution of their zeros as well as on the asymptotic behavior of such polynomials under some restrictions about the measure of orthogonality.

Key Words.
matrix orthogonal polynomials, zeros, asymptotic behavior.

AMS(MOS) Subject Classifications.
42C05, 15A15, 15A23.

Files.
vol.14.2002/pp127-141.dir/pp127-141.ps;
vol.14.2002/pp127-141.dir/pp127-141.pdf;

Forward References.

142 Polynomial inequalities, functional spaces and best approximation on the real semiaxis with Laguerre weights. *G. Mastroianni.*

Abstract.
This is a short survey on polynomial approximation with Laguerre weights. Some new polynomial inequalities are presented.

Key Words.
Orthogonal polynomials, polynomial inequalities, Fourier series, Lagrange interpolation.

AMS(MOS) Subject Classifications.
41A10, 26D05.

Files.
vol.14.2002/pp142-151.dir/pp142-151.ps;
vol.14.2002/pp142-151.dir/pp142-151.pdf;

Forward References.

152 Bounds for Vandermonde type determinants of orthogonal polynomials. *Gerhard Schmeisser.*

Abstract.
Let be a system of monic orthogonal polynomials. We establish upper and lower estimates for determinants of the form For the proofs, we have to study the monic orthogonal system obtained by inserting the polynomial as a weight into the inner

product defining . We also express the recurrence formula for in terms of Vandermonde type determinants.

Key Words.
Vandermonde type determinants, orthogonal systems, polynomial weights, inequalities.

AMS(MOS) Subject Classifications.
42C05, 15A15, 15A45, 30A10.

Files.
vol.14.2002/pp152-164.dir/pp152-164.ps;
vol.14.2002/pp152-164.dir/pp152-164.pdf;

Forward References.

165 Uncertainty principles revisited. *Kathi K. Selig.*

Abstract.
The Heisenberg uncertainty principle and the uncertainty principle for self-adjoint operators have been known and applied for decades. Both in quantum mechanics and in time-frequency analysis they play an important role. In this paper, the uncertainty principle is extended to symmetric operators and to normal operators. Further, different function spaces are studied in which we obtain a number of uncertainty principles of same type using various operators.

Key Words.
uncertainty principle, self-adjoint operators, symmetric operators, normal operators, periodic functions, ultraspherical polynomials, sphere.

AMS(MOS) Subject Classifications.
26D10, 42C25, 47A05, 47A30, 47B47.

Files.
vol.14.2002/pp165-177.dir/pp165-177.ps;
vol.14.2002/pp165-177.dir/pp165-177.pdf;

Forward References.

178 Comparing multilevel coarsening strategies. *Frauke Sprengel.*

Abstract.
We compare several multilevel coarsening strategies by using stable subspace splitting techniques. The obtained condition numbers give an answer on how well the coarsening strategies are suited for solving an anisotropic elliptic boundary value problem.

Key Words.
Finite elements, multilevel algorithms, semi-coarsening.

AMS(MOS) Subject Classifications.
65N30, 65N55, 65N22.

Files.
vol.14.2002/pp178-194.dir/pp178-194.ps;
vol.14.2002/pp178-194.dir/pp178-194.pdf;

Forward References.

195 Asymptotics for quadratic Hermite-Padé polynomials associated with the exponential function. *Herbert Stahl.*

Abstract.
The asymptotic behavior of quadratic Hermite-Padé polynomials p_n, q_n, $r_n \in \mathcal{P}_\backslash$ of type I and $\mathfrak{p}_n$, $\mathfrak{q}_\mathfrak{n}$, $\mathfrak{r}_\mathfrak{n} \in \mathcal{P}_{\in\backslash}$ of type II associated with the exponential function are studied. In the introduction the background of the definition of Hermite-Padé polynomials is reviewed. The quadratic Hermite-Padé polynomials p_n, q_n, $r_n \in \mathcal{P}_\backslash$ of type I are defined by the relation

$$p_n(z) + q_n(z)e^z + r_n(z)e^{2z} \;=\; O(z^{3n+2}) \quad \text{as} \quad z \to 0,$$

and the polynomials $\mathfrak{p}_\mathfrak{n}$, $\mathfrak{q}_\mathfrak{n}$, $\mathfrak{r}_\mathfrak{n} \in \mathcal{P}_{\in\backslash}$ of type II by the two relations

$$\begin{aligned} \mathfrak{p}_\mathfrak{n}(\mathfrak{z})\mathfrak{e}^{\mathfrak{z}} - \mathfrak{q}_\mathfrak{n}(\mathfrak{z}) \;&=\; O(z^{3n+1}) \quad \text{as} \quad z \to 0, \\ \mathfrak{p}_\mathfrak{n}(\mathfrak{z})\mathfrak{e}^{2\mathfrak{z}} - \mathfrak{r}_\mathfrak{n}(\mathfrak{z}) \;&=\; O(z^{3n+1}) \quad \text{as} \quad z \to 0. \end{aligned}$$

Analytic descriptions are given for the arcs, on which the contracted zeros of both sets of the polynomials $\{p_n, q_n, r_n\}$ and $\{\mathfrak{p}_\mathfrak{n}, \mathfrak{q}_\mathfrak{n}, \mathfrak{r}_\mathfrak{n}\}$ cluster as $n \to \infty$. Analytic expressions are also given for the density functions of the asymptotic distributions of these zeros.
The description is based on an algebraic function of third degree and a harmonic function defined on the Riemann surface, which is associated with the algebraic function. The existence and basic properties of the asymptotic distributions of the zeros and the arcs on which these distributions live are proved, the asymptotic relations themselves are only conjectured. Numerical calculations are presented, which demonstrate the plausibility of these conjectures.

Key Words.
Quadratic Hermite-Padé polynomials of type I and type II, the exponential function, German and Latin polynomials, Hermite-Padé approximants.

AMS(MOS) Subject Classifications.
41A21, 30E10.

Files.
vol.14.2002/pp195-222.dir/pp195-222.ps;
vol.14.2002/pp195-222.dir/pp195-222.pdf;

Forward References.

223 On convergence and divergence of Fourier–Bessel series. *Krzysztof Stempak.*

Abstract.
We furnish another proof, based on an idea of Prestini, of a maximal inequality for the partial sum operators of Fourier–Bessel expansions proved by Guadalupe, Pérez, Ruiz and Varona. Divergence results and mean convergence are also discussed.

Key Words.
Fourier–Bessel expansions, almost everywhere and norm convergence.

AMS(MOS) Subject Classifications.
42C10.

Files.
vol.14.2002/pp223-235.dir/pp223-235.ps;
vol.14.2002/pp223-235.dir/pp223-235.pdf;

Forward References.