

BOUNDS FOR VANDERMONDE TYPE DETERMINANTS OF ORTHOGONAL POLYNOMIALS*

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Abstract. Let $(P_n)_{n \in \mathbb{N}_0}$ be a system of monic orthogonal polynomials. We establish upper and lower estimates for determinants of the form

$$V_n(z_1, \dots, z_k) := \det \begin{pmatrix} P_n(z_1) & \dots & P_{n+k-1}(z_1) \\ \vdots & & \vdots \\ P_n(z_k) & \dots & P_{n+k-1}(z_k) \end{pmatrix}.$$

For the proofs, we have to study the monic orthogonal system $(P_n^{[w]})_{n \in \mathbb{N}_0}$ obtained by inserting the polynomial $w(x) := \prod_{\nu=1}^k (x - z_\nu)$ as a weight into the inner product defining $(P_n)_{n \in \mathbb{N}_0}$. We also express the recurrence formula for $(P_n^{[w]})_{n \in \mathbb{N}_0}$ in terms of Vandermonde type determinants.

Key words. Vandermonde type determinants, orthogonal systems, polynomial weights, inequalities.

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