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A QUADRATURE FORMULA OF RATIONAL TYPE FOR INTEGRANDS WITH ONE ENDPOINT SINGULARITY*

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Abstract. The paper deals with the construction of an efficient quadrature formula of rational type to evaluate the integral of functions which are analytic in the interval of integration, except at the endpoints. Basically our approach consists in introducing a change of variable u_q into the integral $I(f,h,r)$

$$I(f,h,r) = \int_{-(1-h)}^{(1-h)r} f(x)dx = \int_{\mu_q}^{\rho_q} F(u_q(x))u_q'(x)dx = I(f,q,h,r),$$

where $f \in H^p$ and $u_q(x) = w_a^q(x) = w_a(w_a^{q-1}(x))$, $w_a(z) = (z-a)/(1-az)$, $0 < a < 1$.

We evaluate the new form $I(f,q,h,r)$ by a quadrature approximant $Q_n(f) = Q(f,n,q,h,r,a)$ which is based on Hermite interpolation by means of rational functions. The nodes of $Q_n(f)$ are derived from a fundamental result proved by Ganelius [Anal. Math., 5 (1979), pp. 19-33] in connection with the problem of approximating the function $f_\alpha(x) = x^\alpha$, $0 \le x \le 1$, by means of rational functions.

We find (a_n) such that $Q_n(f) \to I(f,r) = I(f,0,r)$ as $h_n = \epsilon(1-a_n) \to 0$, for all $f \in H^p$. For functions in H^p, $1 < p < \infty$, which satisfy an integral Lipschitz condition of order β, the following estimate is deduced

$$E_n(f) = |I(f,r) - Q_n(f)| \le M\sqrt{n}\,\exp\left(-\pi\sqrt{n\beta(2q-1-1/p)}\right).$$

If $\beta = q = 1$ then the upper bound for $E_n(f)$ is that which is exact for the optimal quadrature error in H^p, $p > 1$.

We report some numerical examples to illustrate the behavior of the method for several values of the parameters.

Key words. interpolatory quadrature formulas, rational approximation, order of convergence, boundary singularities.

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