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Contents

Abstract.

For a given the -numerical range of an matrix polynomial is defined by . In this paper, an inclusion-exclusion methodology for the estimation of is proposed. Our approach is based on i) the discretization of a region that contains , and ii) the construction of an open circular disk, which does not intersect , centered at every grid point . For the cases and an important difference arises in one of the steps of the algorithm. Thus, these two cases are discussed separately.

Key Words.
matrix polynomial, eigenvalue, -numerical range, boundary, inner -numerical radius, Davis-Wielandt shell.

AMS(MOS) Subject Classifications.
15A22, 15A60, 65D18, 65F30, 65F35.

Files.
/etna/vol.17.2004/pp1-10.dir/pp1-10.ps;
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Forward References.

Abstract.

In this paper we consider polynomial collocation methods for the numerical solution of a singular integral equation over the interval, where the operator of the equation is supposed to be of the form with the Cauchy singular integral operator, with piecewise continuous coefficients and and with a Jacobi weight denotes an integral operator with a continuous kernel function. To the integral equation we apply two collocation methods, where the collocation points are the Chebyshev nodes of the first and second kind and where the trial space is the space of polynomials multiplied by another Jacobi weight. For the stability and convergence of this collocation scheme in weighted -spaces, we derive necessary and sufficient conditions. Moreover, we discuss stability of operator sequences belonging to algebras generated by the sequences of the collocation methods for the above described operators. Finally, the so-called splitting property of the singular values of the sequences of the matrices of the discretized equations is proved.

Key Words.

Cauchy singular integral equation, polynomial collocation method, stability, singular values, splitting property.

AMS(MOS) Subject Classifications.

45L10, 65R20, 65N38.

Files.
/etna/vol.17.2004/pp11-75.dir/pp11-75.ps;
/etna/vol.17.2004/pp11-75.dir/pp11-75.pdf;

Forward References.

76 Strong rank revealing Cholesky factorization. *M. Gu and L. Miranian.*

Abstract.

For any symmetric positive definite matrix we introduce a definition of strong rank revealing Cholesky (RRCh) factorization similar to the notion of strong rank revealing QR factorization developed in the joint work of Gu and Eisenstat. There are certain key properties attached to strong RRCh factorization, the importance of which is discussed by Higham in the context of backward stability in his work on Cholesky decomposition of semi-definite matrices. We prove the existence of a pivoting strategy which, if applied in addition to standard Cholesky decomposition, leads to a strong RRCh factorization, and present two algorithms which use pivoting strategies based on the idea of local maximum volumes to compute a strong RRCh decomposition.

Key Words.
Cholesky decomposition, LU decomposition, QR decomposition, rank revealing, numerical rank, singular values, strong rank revealing QR factorization.

AMS(MOS) Subject Classifications.

65F30.

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/etna/vol.17.2004/pp76-92.dir/pp76-92.ps;
/etna/vol.17.2004/pp76-92.dir/pp76-92.pdf;

Forward References.

93 Asymptotic lower bounds for eigenvalues by nonconforming finite element methods. *María G. Armentano and Ricardo G. Durán.*

Abstract.

We analyze the approximation obtained for the eigenvalues of the Laplace operator by the nonconforming piecewise linear finite element of Crouzeix-Raviart. For singular eigenfunctions, as those arising in nonconvex polygons, we prove that the eigenvalues obtained with this method give lower bounds of the exact eigenvalues when the mesh size is small enough.

Key Words.

finite elements, eigenvalue problems, nonconforming methods.

AMS(MOS) Subject Classifications.

65N25, 65N30.

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/etna/vol.17.2004/pp93-101.dir/pp93-101.ps;
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Forward References.

102 On the existence theorems of Kantorovich, Miranda and Borsuk. *Götz Alefeld, Andreas Frommer, Gerhard Heindl and Jan Mayer.*

Abstract.

The theorems of Kantorovich, Miranda and Borsuk all give conditions on the existence of a zero of a nonlinear mapping. In this paper we are concerned with relations between these theorems in terms of generality in the case that the mapping is finite-dimensional. To this purpose we formulate a generalization of Miranda's theorem, holding for arbitrary norms instead of just the -norm. As our main results we then prove that the Kantorovich theorem reduces to a special case of this generalized Miranda theorem as well as to a special case of Borsuk's theorem. Moreover, it turns out that, essentially, the Miranda theorems are themselves special cases of Borsuk's theorem.

Key Words.

nonlinear equations, existence theorems, fixed points, Newton-Kantorovich theorem, Miranda theorem, Borsuk theorem.

AMS(MOS) Subject Classifications.

47H10, 47J05, 65H10.

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/etna/vol.17.2004/pp102-111.dir/pp102-111.ps;
/etna/vol.17.2004/pp102-111.dir/pp102-111.pdf;

Forward References.

112 Convergence of V-cycle and F-cycle multigrid methods for the biharmonic problem using the Morley element. *Jie Zhao.*

Abstract.

Multigrid V-cycle and F-cycle algorithms for the biharmonic problem using the Morley element are studied in this paper. We show that the contraction numbers can be uniformly improved by increasing the number of smoothing steps.

Key Words.

multigrid, nonconforming, V-cycle, F-cycle, biharmonic problem, Morley element.

AMS(MOS) Subject Classifications.

65N55, 65N30.

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/etna/vol.17.2004/pp112-132.dir/pp112-132.ps;
/etna/vol.17.2004/pp112-132.dir/pp112-132.pdf;

Forward References.

133 Quadrature of singular integrands over surfaces. *Kendall Atkinson.*

Abstract.

Consider integration over a simple closed smooth surface in , one that is homeomorphic to the unit sphere, and suppose the integrand has a point singularity. We propose a numerical integration method based on using transformations that lead to an integration problem over the unit sphere with an integrand that is much smoother. At this point, the trapezoidal rule is applied to the spherical coordinate representation of the problem. The method is simple to apply and it results in rapid convergence. The intended application is to the evaluation of boundary integrals arising in boundary integral equation methods in potential theory and the radiosity equation.

Key Words.

spherical integration, singular integrand, boundary integral, trapezoidal rule.

AMS(MOS) Subject Classifications.

65D32, 65B15.

Files.
/etna/vol.17.2004/pp133-150.dir/pp133-150.ps;
/etna/vol.17.2004/pp133-150.dir/pp133-150.pdf;

Forward References.

151 A quadratically convergent Bernoulli-like algorithm for solving matrix polynomial equations in Markov chains. *C. He, B. Meini, N.H. Rhee and K. Sohraby.*

Abstract.

A quadratically convergent algorithm is developed for solving matrix polynomial equations arising in M/G/1 and G/M/1 type Markov chains. The algorithm is based on the computation of generalized block eigenvalues/vectors of a suitable pair of matrices by means of a Bernoulli-like method. The use of the displacement structure allows one to reduce the computational cost per step. A shifting technique speeds up the rate of convergence.

Key Words.

polynomial matrix equations, Markov chains, generalized eigenvalues/eigenvectors, displacement structure.

AMS(MOS) Subject Classifications.

15A24, 60J22, 65F15.

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/etna/vol.17.2004/pp151-167.dir/pp151-167.ps;
/etna/vol.17.2004/pp151-167.dir/pp151-167.pdf;

Forward References.

168 Multidimensional smoothing using hyperbolic interpolatory wavelets. *M. Hegland, O. M. Nielsen and Z. Shen.*

Abstract.

We propose the application of hyperbolic interpolatory wavelets for large-scale d-dimensional data fitting. In particular, we show how wavelets can be used as a highly efficient tool for multidimensional smoothing. The grid underlying these wavelets is a sparse grid. The hyperbolic interpolatory wavelet space of level j uses $O(j^{d-1}2^j)$ basis functions and it is shown that under sufficient smoothness an approximation error of order $O\left(\binom{j+d-1}{d-1}2^{-2j}\right)$ can be achieved. The implementation uses the fast wavelet transform and an efficient indexing method to access the wavelet coefficients. A practical example demonstrates the efficiency of the approach.

Key Words.

sparse grids, predictive modelling, wavelets, smoothing, data mining.

AMS(MOS) Subject Classifications.

65C60, 65D10, 65T60.

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/etna/vol.17.2004/pp168-180.dir/pp168-180.ps;
/etna/vol.17.2004/pp168-180.dir/pp168-180.pdf;

Forward References.

181 A frequency decomposition waveform relaxation algorithm for semilinear evolution equations. *M. J. Gander.*

Abstract.

Semilinear evolution equations arise in many applications ranging from mathematical biology to chemical reactions (e.g., combustion). The significant difficulty in these equations is the nonlinearity, which combined with the discretized diffusion operator leads to large systems of nonlinear equations. To solve these equations, Newton's method or a variant thereof is often used, and to achieve convergence can require individual fine tuning for each case. This can be especially difficult if nothing is known about the solution behavior. In addition, one observes in many cases that not all frequency components are equally important for the solution; the frequency interaction is determined by the nonlinearity. It is therefore of interest to

work in frequency space when analyzing the unknown behavior of such problems numerically.
We propose in this paper an algorithm which reduces the dimensionality of the nonlinear problems to be solved to a size chosen by the user. The algorithm performs a decomposition in frequency space into subspaces, and an iteration is used to obtain the solution of the original problem from the solutions on the frequency subspaces. We prove linear convergence of the algorithm on unbounded time intervals, a result which is also valid for the stationary case. On bounded time intervals, we show that the new algorithm converges superlinearly, a rate faster than any linear rate. We obtain this result by relating the algorithm to an algorithm of waveform relaxation type. By using time windows, one can thus achieve any linear contraction rate desired. An additional advantage of this algorithm is its inherent parallelism.

Key Words.

waveform relaxation, frequency decomposition, sequential spectral method, iterative approximation of evolution problems.

AMS(MOS) Subject Classifications.

65M70, 65M55, 65H10.

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/etna/vol.17.2004/pp181-194.dir/pp181-194.ps;
/etna/vol.17.2004/pp181-194.dir/pp181-194.pdf;

Forward References.

195 Improved initialization of the accelerated and robust QR-like polynomial root-finding. *D.A. Bini, L. Gemignani, V. Y. Pan.*

Abstract.

We approximate polynomial roots numerically as the eigenvalues of a unitary diagonal plus rank-one matrix. We rely on our earlier adaptation of the QR algorithm, which exploits the semiseparable matrix structure to approximate the eigenvalues in a fast and robust way, but we substantially improve the performance of the resulting algorithm at the initial stage, as confirmed by our numerical tests.

Key Words.

QR iteration, eigenvalue computation, polynomial roots, semiseparable matrices, DFT, FFT, Moebius transformation.

AMS(MOS) Subject Classifications.

65H17, 65F15.

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/etna/vol.17.2004/pp195-205.dir/pp195-205.ps;
/etna/vol.17.2004/pp195-205.dir/pp195-205.pdf;

Forward References.

206 On the numerical solution of some semilinear elliptic problems. *K. Atkinson and W. Han.*

Abstract.

We discuss a general framework for the numerical solution of a family of semilinear elliptic problems whose leading differential operator is the Laplacian. A problem is first transformed to one on a standard domain via a conformal mapping. The boundary value problem on the standard domain is then reduced to an equivalent integral operator equation. We employ the Galerkin method to solve the integral operator equation, using the eigenfunctions of the Laplacian on the standard domain. An error analysis of the method is given.

Key Words.

elliptic, nonlinear, integral equation, Galerkin method.

AMS(MOS) Subject Classifications.

65R20, 65N99, 35J65.

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/etna/vol.17.2004/pp206-217.dir/pp206-217.ps;
/etna/vol.17.2004/pp206-217.dir/pp206-217.pdf;

Forward References.

218 A note on the efficiency of residual-based a-posteriori error estimators for some mixed finite element methods. *G. N. Gatica.*

Abstract.

In this paper we present a unified proof of the efficiency of residual-based a-posteriori error estimates for the dual-mixed variational formulations of linear boundary value problems in the plane. We consider the interior problem determined by a second order elliptic equation in divergence form with mixed boundary conditions, and the exterior transmission problem given by the same equation in a bounded domain, coupled with Laplace equation in the surrounding unbounded exterior region. The corresponding Galerkin scheme reduces to a mixed finite element method with Lagrange multipliers for the first problem, and to the coupling of the mixed finite element method with the boundary element method for the second one. Our analysis makes use of inverse inequalities in finite element subspaces and the localization technique based on triangle-bubble and edge-bubble functions.

Key Words.

mixed finite elements, boundary elements, residual-based estimates, efficiency.

AMS(MOS) Subject Classifications.

65N15, 65N30, 65N50, 35J25.

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/etna/vol.17.2004/pp218-233.dir/pp218-233.ps;
/etna/vol.17.2004/pp218-233.dir/pp218-233.pdf;

Forward References.