

ON THE ESTIMATION OF THE Q -NUMERICAL RANGE OF MONIC MATRIX POLYNOMIALS*

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Abstract. For a given $q \in [0, 1]$, the q -numerical range of an $n \times n$ matrix polynomial $P(\lambda) = I\lambda^m + A_{m-1}\lambda^{m-1} + \dots + A_1\lambda + A_0$ is defined by $W_q(P) = \{\lambda \in \mathbb{C} : y^*P(\lambda)x = 0, x, y \in \mathbb{C}^n, x^*x = y^*y = 1, y^*x = q\}$. In this paper, an inclusion-exclusion methodology for the estimation of $W_q(P)$ is proposed. Our approach is based on i) the discretization of a region Ω that contains $W_q(P)$, and ii) the construction of an open circular disk, which does not intersect $W_q(P)$, centered at every grid point $\mu \in \Omega \setminus W_q(P)$. For the cases $q = 1$ and $0 < q < 1$, an important difference arises in one of the steps of the algorithm. Thus, these two cases are discussed separately.

Key words. matrix polynomial, eigenvalue, q -numerical range, boundary, inner q -numerical radius, Davis-Wielandt shell.

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