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Contents

Abstract.

The message of this paper is that powers of large Toeplitz matrices show critical behavior if and only if the L^∞ norm of the symbol is greater than one. Critical behavior means that the norms of the powers grow exponentially to infinity or that they run through a critical transient phase before decaying exponentially to zero. We summarize several known results that are relevant to the problem. Moreover, the paper contains some new results and illuminates the question from a perspective that might be new. The paper may serve as an introduction to a few basic phenomena one encounters in the field.

Key Words.
Toeplitz matrix, matrix power, matrix exponential, transient behavior.

AMS(MOS) Subject Classifications.
47B35, 15A60, 65F35.

Files.
/etna/vol.18.2004/pp1-41.dir/pp1-41.ps;
/etna/vol.18.2004/pp1-41.dir/pp1-41.pdf.

Forward References.

Abstract.

Discrete Sobolev and Poincaré inequalities are derived for piecewise polynomial functions on two dimensional domains. These inequalities can be applied to classical nonconforming finite element methods and discontinuous Galerkin methods.

Key Words.
discrete Sobolev inequality, discrete Poincaré inequality, piecewise polynomial functions, nonconforming, discontinuous Galerkin.

AMS(MOS) Subject Classifications.
65N30

Files.
/etna/vol.18.2004/pp42-48.dir/pp42-48.ps;
/etna/vol.18.2004/pp42-48.dir/pp42-48.pdf.

Files.
/etna/vol.18.2004/pp65-72.dir/pp65-72.ps;
/etna/vol.18.2004/pp65-72.dir/pp65-72.pdf.

Forward References.

73 A New Geršgorin-Type Eigenvalue Inclusion Set. *Ljiljana Cvetkovic, Vladimir Kostic, and Richard S. Varga.*

Abstract.

We give a generalization of a less well-known result of Dashnic and Zusmanovich from 1970, and show how this generalization compares with related results in this area.

Key Words.
Geršgorin theorem, Brauer Cassini ovals, nonsingularity results.

AMS(MOS) Subject Classifications.
15A18, 65F15.

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/etna/vol.18.2004/pp73-80.dir/pp73-80.ps;
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Forward References.

81 Some theoretical results derived from polynomial numerical hulls of Jordan blocks. *Anne Greenbaum.*

Abstract.

The polynomial numerical hull of degree k for a square matrix A is a set in the complex plane designed to give useful information about the norms of functions of the matrix; it is defined as

$$\{z \in \mathbb{C} : \ \|p(A)\| \geq |p(z)| \ \text{ for all polynomials } p \text{ of degree } k \text{ or less}\}.$$

In a previous paper [V. Faber, A. Greenbaum, and D. Marshall, *The polynomial numerical hulls of Jordan blocks and related matrices*, Linear Algebra Appl., 374 (2003), pp. 231–246] analytic expressions were derived for the polynomial numerical hulls of Jordan blocks. In this paper, we explore some consequences of these results. We derive lower bounds on the norms of functions of Jordan blocks and triangular Toeplitz matrices that approach equalities as the matrix size approaches infinity. We demonstrate that even for moderate size matrices these bounds give fairly good estimates of the behavior of matrix powers, the matrix exponential, and the resolvent norm. We give new estimates of the convergence rate of the GMRES algorithm applied to a Jordan block. We also derive a new estimate for the field of values of a general Toeplitz matrix.

Key Words.
polynomial numerical hull, field of values, Toeplitz matrix.

AMS(MOS) Subject Classifications.
15A60, 65F15, 65F35.

Abstract.

For a given matrix A we compute the matrix exponential e^{tA} under the assumption that the eigenvalues of A are known, but without determining the eigenvectors. The presented approach exploits the connection between matrix exponentials and confluent Vandermonde matrices V. This approach and the resulting methods are very simple and can be regarded as an alternative to the Jordan canonical form methods. The discussed inversion algorithms for V as well as the matrix representation of V^{-1} are of independent interest also in many other applications.

Key Words.
matrix exponential, Vandermonde matrix, fast algorithm, inverse.

AMS(MOS) Subject Classifications.
34A30, 65F05, 15A09, 15A23.

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/etna/vol.18.2004/pp91-100.dir/pp91-100.ps;
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Forward References.

Abstract.

The monic Jacobi matrix is a tridiagonal matrix which contains the parameters of the three-term recurrence relation satisfied by the sequence of monic polynomials orthogonal with respect to a measure. Darboux transformation without parameter changes a monic Jacobi matrix associated with a measure μ into the monic Jacobi matrix associated with $xd\mu$. This transformation has been used in several numerical problems as in the computation of Gaussian quadrature rules. In this paper, we analyze the stability of an algorithm which implements Darboux transformation without parameter numerically and we also study the sensitivity of the problem. The main result of the paper is that, although the algorithm for Darboux transformation without parameter is not backward stable, it is forward stable. This means that the forward errors are of similar magnitude to those produced by a backward stable algorithm. Moreover, bounds for the forward errors computable with low cost are presented. We also apply the results to some classical families of orthogonal polynomials.

Key Words.
Darboux transformation, orthogonal polynomials, stability, sensitivity, tridiagonal matrices, LU factorization, LR algorithm.

AMS(MOS) Subject Classifications.
65G50, 42C05, 15A23, 65F30, 65F35.

Files.
/etna/vol.18.2004/pp101-136.dir/pp101-136.ps;
/etna/vol.18.2004/pp101-136.dir/pp101-136.pdf.

Forward References.
vol.13.2002/pp119-147.dir/pp119-147.html

Abstract.

We show that the shifted QR iteration applied to a companion matrix F maintains the weakly semiseparable structure of F. More precisely, if $A_i - \alpha_i I = Q_i R_i$, $A_{i+1} := R_i Q_i + \alpha_i I$, $i = 0, 1, \ldots$, where $A_0 = F$, then we prove that Q_i, R_i and A_i are semiseparable matrices having semiseparability rank at most 1, 4 and 3, respectively. This structural property is used to design an algorithm for performing a single step of the QR iteration in just $O(n)$ flops. The robustness and reliability of this algorithm is discussed. Applications to approximating polynomial roots are shown.

Key Words.
companion matrices, QR factorization, QR iteration, semiseparable matrices, eigenvalues, polynomial roots.

AMS(MOS) Subject Classifications.
65F15, 15A18, 65H17.

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/etna/vol.18.2004/pp137-152.dir/pp137-152.ps;
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Forward References.

Abstract.

Many numerical methods for the solution of ill-posed problems are based on Tikhonov regularization. Recently, Rojas and Steihaug described a barrier method for computing nonnegative Tikhonov-regularized approximate solutions of linear discrete ill-posed problems. Their method is based on solving a sequence of parameterized eigenvalue problems. This paper describes how the solution of parametrized eigenvalue problems can be avoided by computing bounds that follow from the connection between the Lanczos process, orthogonal polynomials and Gauss quadrature.

Key Words.
ill-posed problem, inverse problem, solution constraint, Lanczos methods, Gauss quadrature.

Abstract.

We present, for some classes of matrices, LDU-decompositions whose unit triangular factors L and U are simultaneously very well conditioned. Our examples include diagonally dominant matrices by rows and columns and their inverses, Stieljes matrices and M-matrices diagonally dominant by rows or columns. We also show a construction of an accurate computation of the LDU-decomposition for any M-matrix diagonally dominant by rows or columns, which in turn can be applied to obtain an accurate singular value decomposition.

Key Words.
conditioning, diagonal dominance, pivoting strategies, accuracy, singular value decomposition.

AMS(MOS) Subject Classifications.
65F05, 65F35, 15A12.