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ON THE SHIFTED QR ITERATION APPLIED TO COMPANION MATRICES*

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Abstract. We show that the shifted QR iteration applied to a companion matrix F maintains the weakly semiseparable structure of F. More precisely, if $A_i - \alpha_i I = Q_i R_i$, $A_{i+1} := R_i Q_i + \alpha_i I$, $i = 0, 1, \ldots$, where $A_0 = F$, then we prove that Q_i, R_i and A_i are semiseparable matrices having semiseparability rank at most 1, 4 and 3, respectively. This structural property is used to design an algorithm for performing a single step of the QR iteration in just $O(n)$ flops. The robustness and reliability of this algorithm is discussed. Applications to approximating polynomial roots are shown.

Key words. companion matrices, QR factorization, QR iteration, semiseparable matrices, eigenvalues, polynomial roots.

AMS subject classifications. 65F15, 15A18, 65H17.

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