

## IMPLICIT FOR LOCAL EFFECTS AND EXPLICIT FOR NONLOCAL EFFECTS IS UNCONDITIONALLY STABLE\*

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**Abstract.** A combination of implicit and explicit timestepping is analyzed for a system of ODEs motivated by ones arising from spatial discretizations of evolutionary partial differential equations. Loosely speaking, the method we consider is implicit in local and stabilizing terms in the underlying PDE and explicit in nonlocal and unstabilizing terms. Unconditional stability and convergence of the numerical scheme are proved by the energy method and by algebraic techniques. This stability result is surprising because usually when different methods are combined, the stability properties of the least stable method plays a determining role in the combination.

Key words. unconditional stability, implicit-explicit methods, multiscale integration.

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