ORTHOGONALITY OF JACOBI POLYNOMIALS WITH GENERAL PARAMETERS[*]

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Abstract. In this paper we study the orthogonality conditions satisfied by Jacobi polynomials $P_n^{(\alpha,\beta)}$ when the parameters α and β are not necessarily > -1. We establish orthogonality on a generic closed contour on a Riemann surface. Depending on the parameters, this leads to either full orthogonality conditions on a single contour in the plane, or to multiple orthogonality conditions on a number of contours in the plane. In all cases we show that the orthogonality conditions characterize the Jacobi polynomial $P_n^{(\alpha,\beta)}$ of degree n up to a constant factor.

Key words. Jacobi polynomials, orthogonality, Rodrigues formula, zeros.

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