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ETNA
Kent State University
etna@mcs.kent.edu

CONVERGENCE OF INFINITE PRODUCTS OF MATRICES AND INNER–OUTER ITERATION SCHEMES*

RAFAEL BRU†, L. ELSNER‡, AND M. NEUMANN§

Dedicated to Wilhelm Niethammer on the occasion of his sixtieth birthday.

Abstract. We develop conditions under which a product $\prod_{i=0}^{\infty} T_i$ of matrices chosen from a possibly infinite set of matrices $\mathcal{S} = \{T_j | j \in J\}$ converges. We obtain the following conditions which are sufficient for the convergence of the product: There exists a vector norm such that all matrices in $\mathcal{S}$ are nonexpansive with respect to this norm and there exists a subsequence $\{i_k\}_{k=0}^{\infty}$ of the sequence of the nonnegative integers such that the corresponding sequence of operators $\left\{T_{i_k}\right\}_{k=0}^{\infty}$ converges to an operator which is paracontracting with respect to this norm. We deduce the continuity of the limit of the product of matrices as a function of the sequences $\{i_k\}_{k=0}^{\infty}$. But more importantly, we apply our results to the question of the convergence of inner–outer iteration schemes for solving **singular** consistent linear systems of equations, where the outer splitting is regular and the inner splitting is weak regular.

Key words. iterative methods, infinite products, contractions.

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† Departament de Matemàtica Aplicada, Universitat Politècnica de València, 46071 València, Spain. This research was supported by grants CICYT #TIC91–1157–C03–01 DGICYT and #PB91–0535.

‡ Fakultät für Mathematik, Universität Bielefeld, Postfach 100131, 33501–Bielefeld, Federal Republic of Germany. This research was supported by Sonderforschungsbereich 343. "*Diskrete Strukturen in der Mathematik*", Universität Bielefeld.

§ Department of Mathematics, University of Connecticut, Storrs, Connecticut 06269–3009, USA. The research was supported by NSF Grants Nos. DMS–9007030 and DMS–9306357 and by Sonderforschungsbereich 343 "*Diskrete Strukturen in der Mathematik*", Universität Bielefeld.