

QUADRATURE OVER THE SPHERE *

KENDALL ATKINSON[†] AND ALVISE SOMMARIVA[‡]

Abstract. Consider integration over the unit sphere in \mathbb{R}^3 , especially when the integrand has singular behaviour in a polar region. In an earlier paper [4], a numerical integration method was proposed that uses a transformation that leads to an integration problem over the unit sphere with an integrand that is much smoother in the polar regions of the sphere. The transformation uses a *grading parameter* q . The trapezoidal rule is applied to the spherical coordinates representation of the transformed problem. The method is simple to apply, and it was shown in [4] to have convergence $O(h^{2q})$ or better for integer values of $2q$. In this paper, we extend those results to non-integral values of $2q$. We also examine superconvergence that was observed when $2q$ is an odd integer. The overall results agree with those of [11], although the latter is for a different, but related, class of transformations.

Key words. spherical integration, trapezoidal rule, Euler-MacLaurin expansion

AMS subject classifications. 65D32

* Received June 8, 2004. Accepted for publication March 15, 2005. Recommended by F. Stenger.

[†] Depts of Mathematics and Computer Science, University of Iowa

[‡] Dept of Pure and Applied Mathematics, University of Padua. Supported by the research project CPDA028291 "Efficient approximation methods for nonlocal discrete transform" of the University of Padua.