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RECURSIVE COMPUTATION OF CERTAIN INTEGRALS OF ELLIPTIC TYPE*

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Abstract. An algorithm for the numerical calculation of the integral function

$$N_n(x) = \int_0^{\pi/2} \frac{\cos^{2n}(\Phi)}{\sqrt{1-x\cdot\sin^2(\Phi)}} \cdot d\Phi \quad (0 \leq x < 1;\ n = 0, 1, 2, \ldots),$$

distinguished solution of the second-order difference equation

$$(2n+1)\cdot x \cdot N_{n+1}(x) + 2n \cdot (1-2x) \cdot N_n(x) = (2n-1)\cdot(1-x)\cdot N_{n-1}(x) \quad (n = 1, 2, \ldots),$$

that uses the recurrence relation and its related continued fraction expansion, is described and discussed. The numerical efficiency of the algorithm is analysed for various x values of the interval $(0 \leq x < 1)$. A twelve digits tabulation of $N_n(x)$ for $n = 1(1)20$ and $x = 0(0.02)1$ is presented as example of the algorithm utilization.

Key words. recurrence relations, elliptic integrals, continued fractions

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