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STABILITY OF NUMERICAL METHODS FOR ORDINARY STOCHASTIC DIFFERENTIAL EQUATIONS ALONG LYAPUNOV-TYPE AND OTHER FUNCTIONS WITH VARIABLE STEP SIZES*

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Abstract. Some general concepts and theorems on the stability of numerical methods for ordinary stochastic differential equations (SDEs) along Lyapunov-type and other Borel-measurable, nonnegative functions are presented. In particular, we deal with almost sure, moment and weak V-stability, exponential and asymptotic stability of related stochastic difference equations with nonrandom, variable step sizes. The applicability of the main results is explained with the class of balanced implicit methods (i.e. certain stochastic linear-implicit methods with appropriate weights). It is shown that, they are rich enough to provide asymptotically, exponentially and polynomially stable numerical methods discretizing stable continuous time SDEs by controlling the choice of their weights.

Key words. stochastic-numerical approximation, stochastic stability, ordinary stochastic differential equations, numerical methods, drift-implicit Euler methods, balanced implicit methods, Lyapunov-type functions, numerical weak V-stability, stability of moments, a.s. stability, asymptotic stability

AMS subject classifications. 65C20, 65C30, 65C50, 60H10, 37H10, 34F05

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