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QR FACTORIZATIONS USING A RESTRICTED SET OF ROTATIONS*

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Dedicated to Alan George on the occasion of his 60th birthday

Abstract. Any matrix A of dimension $m \times n$ ($m \geq n$) can be reduced to upper triangular form by multiplying by a sequence of $mn - n(n+1)/2$ appropriately chosen rotation matrices. In this work, we address the question of whether such a factorization exists when the set of allowed rotation planes is restricted. We introduce the rotation graph as a tool to devise elimination orderings in QR factorizations. Properties of this graph characterize sets of rotation planes that are sufficient (or sufficient under permutation) and identify rotation planes to add to complete a deficient set. We also devise a constructive way to determine all feasible rotation sequences for performing the QR factorization using a restricted set of rotation planes. We present applications to quantum circuit design and parallel QR factorization.

Key words. QR decomposition, Givens rotations, plane rotations, parallel QR decomposition, quantum circuit design, qudits

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