

COMBINATORIAL ALGORITHMS FOR COMPUTING COLUMN SPACE BASES THAT HAVE SPARSE INVERSES*

ALI PINAR[†], EDMOND CHOW[‡], AND ALEX POTHEN[§]

Abstract. This paper presents a new combinatorial approach towards constructing a sparse, implicit basis for the null space of a sparse, under-determined matrix A . Our approach is to compute a column space basis of A that has a sparse inverse, which could be used to represent a null space basis in implicit form. We investigate three different algorithms for computing column space bases: two greedy algorithms implemented using graph matchings, and a third, which employs a divide and conquer strategy implemented with hypergraph partitioning followed by a matching. Our results show that for many matrices from linear programming, structural analysis, and circuit simulation, it is possible to compute column space bases having sparse inverses, contrary to conventional wisdom. The hypergraph partitioning method yields sparser basis inverses and has low computational time requirements, relative to the greedy approaches. We also discuss the complexity of selecting a column space basis when it is known that such a basis exists in block diagonal form with a given small block size.

Key words. sparse column space basis, sparse null space basis, block angular matrix, block diagonal matrix, matching, hypergraph partitioning, inverse of a basis

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[†]Computational Research Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720 (apinar@lbl.gov). Supported by the Director, Office of Science, Division of Mathematical, Information, and Computational Sciences of the U.S. Department of Energy under contract DE-AC03-76SF00098.

[‡]Center for Applied Scientific Computing, Lawrence Livermore National Laboratory, L-560, Box 808, Livermore, CA 94568. The work of this author was performed under the auspices of the U.S. Department of Energy by University of California Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48. Current Address: D. E. Shaw Research and Development, 39th Floor, 120 West 45th St., New York NY 10036 (etchow@gmail.com).

[§]Computer Science Department and Center for Computational Science, Old Dominion University, Norfolk, VA 23529 (pothen@cs.odu.edu). The work of this author was supported by NSF grants ACI-023722 and CCF 0515218, by DOE grant DE-FC02-01ER25476, and by Lawrence Livermore National Laboratory under contract B542604.