Electronic Transactions on Numerical Analysis.
Volume 22, pp. 71-96, 2006.

ISSN 1068-9613.

ETNA
Kent State University
etna@mcs.kent.edu

AN AUGMENTED LAGRANGIAN APPROACH TO THE NUMERICAL SOLUTION OF THE DIRICHLET PROBLEM FOR THE ELLIPTIC MONGE-AMPÈRE EQUATION IN TWO DIMENSIONS*

E. J. DEAN† AND R. GLOWINSKI‡

Abstract. In this article, we discuss the numerical solution of the Dirichlet problem for the real elliptic Monge-Ampère equation, in two dimensions, by an augmented Lagrangian based iterative method. To derive the above algorithm, we take advantage of a reformulation of the Monge-Ampère problem as a saddle-point one, for a well-chosen augmented Lagrangian functional over the product of appropriate primal and dual sets. The convergence of the finite element approximation and of the iterative methods described in this article still has to be proved, however, on the basis of numerical experiments reported in this article, it is safe to say that: (i) The augmented Lagrangian methodology discussed here provides a sequence converging to a solution of the Monge-Ampère problem under consideration, if such a solution exists in the space $H^2(\Omega)$. (ii) If, despite the smoothness of its data, the above problem has no solution, the above augmented Lagrangian method solves it in a least-squares sense, to be precisely defined in this article.

Key words. elliptic Monge-Ampère equation, augmented Lagrangian algorithms, mixed finite element approximations

AMS subject classifications. 35J60, 65F10, 65N30

*Received September 15, 2004. Accepted for publication February 24, 2005. Recommended by R. Lehoucq. This work was supported in part by the National Science Foundation under grant DMS-0412267.

†Department of Mathematics, University of Houston, 4800 Calhoun Road, Houston, TX 77204-3008 (dean@math.uh.edu).

‡Department of Mathematics, University of Houston, 4800 Calhoun Road, Houston, TX 77204-3008 (roland@math.uh.edu).