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ETNA
Kent State University
etna@mcs.kent.edu

ON EXTREMAL PROBLEMS RELATED TO INVERSE BALAYAGE*

MARIO GÖTZ†

Dedicated to the memory of Professor Aurel Cornea

Abstract. Suppose G is a body in $\mathbb{R}^d$, $D \subset G$ is compact, and ρ a unit measure on ∂G. Inverse balayage refers to the question of whether there exists a measure ν supported inside D such that ρ and ν produce the same electrostatic field outside G. Establishing a duality principle between two extremal problems, it is shown that such an inverse balayage exists if and only if

$$\sup_{\mu} \left\{ \inf_{y \in D} U^{\mu}(y) - \int U^{\rho} \, d\mu \right\} = 0 ,$$

where the supremum is taken over all unit measures μ on ∂G and U^{μ} denotes the electrostatic potential of μ. A consequence is that pairs (ρ, D) admitting such an inverse balayage can be characterized by a ρ-mean-value principle, namely,

$$\sup_{z \in D} h(z) \geq \int h \, d\rho \geq \inf_{z \in D} h(z)$$

for all h harmonic in G and continuous up to the boundary.

In addition, two approaches for the construction of an inverse balayage related to extremal point methods are presented, and the results are applied to problems concerning the determination of restricted Chebychev constants in the theory of polynomial approximation.

Key words. Logarithmic potential, Newtonian potential, balayage, inverse balayage, linear optimization, duality, Chebychev constant, extremal problem.

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†Mathematisch-Geographische Fakultät, Katholische Universität Eichstätt-Ingolstadt, Ostenstraße 26, 85071 Eichstätt, Germany (mario.goetz@ku-eichstaett.de).