Electronic Transactions on Numerical Analysis. Volume 23, pp. 329-338, 2006. Copyright © 2006, Kent State University. ISSN 1068-9613. ETNA Kent State University etna@mcs.kent.edu

## NUMERICAL COMPUTATION OF THE EIGENVALUES FOR THE SPHEROIDAL WAVE EQUATION WITH ACCURATE ERROR ESTIMATION BY MATRIX METHOD\*

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**Abstract.** A method to compute the eigenvalues of the spheroidal wave equations is proposed, as an application of a theorem on eigenvalues of certain classes of infinite matrices. The computation of its inverse problem (namely, solving another parameter  $c^2$  for given eigenvalue  $\lambda$ ) is likewise given. As a result, precise and explicit error estimates are obtained for the approximated eigenvalues.

**Key words.** spheroidal wave equation, eigenvalue, numerical computation, error estimate, infinite symmetric tridiagonal matrix

## AMS subject classifications. 34L16

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