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Contents

1 Orthogonal least squares solutions for linear operators. *Eva Acosta*.

Abstract.
This paper solves the problem of finding, in a least squares sense, the coefficients of a series expansion of a function in terms of a chosen orthogonal basis from the knowledge not of the function itself but from the action of a linear operator upon it. The coefficiens are evaluated by inner product with a set of functions related to the orthogonal basis through the adjoint operator of the linear operator. Examples for both differential operators and integral ones as well as related properties are given.

Key Words.
orthogonal polynomials, linear operators, gradient operator, Radon transform

AMS(MOS) Subject Classifications.
33C90, 33C47,42C05,42C15, 47A05

7 On recurrence relations for radial wave functions for the N-th dimensional oscillators and hydrogenlike atoms: analytical and numerical study. *R. Álvarez-Nodarse, J. L. Cardoso, and N. R. Quintero*.

Abstract.
Using a general procedure for finding recurrence relations for hypergeometric functions and polynomials introduced by Cardoso et al. [J. Phys. A, 36 (2003), pp. 2055-2068] we obtain some new recurrence relations for the radial wave functions of the N-th dimensional isotropic harmonic oscillators as well as the hydrogenlike atoms. A numerical analysis of such recurrences is also presented.

Key Words.
wave functions, linear recurrence relations, Laguerre polynomials

AMS(MOS) Subject Classifications.
33C45, 11B37, 81V45, 81Q99

24 Quantum algebras $su_q(2)$ and $su_q(1,1)$ associated with certain q-Hahn polynomials: a revisited approach. *Jorge Arvesú*.

Abstract.
This contribution deals with the connection of q-Clebsch-Gordan coefficients (q-CGC) of the Wigner-Racah algebra for the quantum groups $SU_q(2)$ and $SU_q(1,1)$ with certain q-Hahn polynomials. A comparative analysis of the properties of these polynomials and $su_q(2)$ and $su_q(1,1)$ Clebsch-Gordan coefficients shows that each relation for q-Hahn polynomials has the corresponding partner among the properties of q-CGC and vice versa. Consequently, special emphasis is given to the calculations carried out in the linear space of polynomials, i.e., to the main characteristics and properties for the new q-Hahn polynomials obtained here by using

the Nikiforov-Uvarov approach [A. F. NIKIFOROV, S. K. SUSLOV AND V. B. UVAROV, *Orthogonal Polynomials in Discrete Variables*, Springer-Verlag, Berlin, 1991; A. F. NIKIFOROV AND V. B. UVAROV, *Classical orthogonal polynomials in a discrete variable on non-uniform lattices*, Preprint Inst. Prikl. Mat. M. V. Keldysh Akad. Nauk SSSR (In Russian), 17, Moscow, 1983] on the non-uniform lattice $x(s) = \frac{q^s-1}{q-1}$. These characteristics and properties will be important to extend the q-Hahn polynomials to the multiple case [J. ARVESÚ, *q-Discrete Multiple Orthogonal Polynomials*, in preparation]. On the other hand, the aforementioned lattice allows to recover the linear one $x(s) = s$ as a limiting case, which doesn't happen in other investigated cases [C. CAMPIGOTTO, YU. F. SMIRNOV AND S. G. ENIKEEV, *q-Analogue of the Kravchuk and Meixner orthogonal polynomials*, J. of Comput. and Appl. Math., 57 (1995), pp. 87–97; A. DEL SOL MESA AND YU. F. SMIRNOV, *Clebsch-Gordan and Racah coefficients for $U_q(1,1)$ quantum algebra*, (Discrete series) In Scattering, Reactions, Transitions in Quantum Systems and Symmetry Methods, R.M. Asherova and Yu.F. Smirnov, eds., 1991], for example in $x(s) = q^{2s}$. This fact suggests that the q-analogues presented here (both from the point of view of quantum group theory and special function theory) are *'good'* ones since all characteristics and properties, and consequently, all matrix element relations will converge to the standard ones when q tends to 1.

Key Words.
Clebsch-Gordan coefficients, discrete orthogonal polynomials (q-discrete orthogonal polynomials), Nikiforov-Uvarov approach, quantum groups and algebras

AMS(MOS) Subject Classifications.
81R50, 33D45, 33C80

45 *Nicolae Cotfas.* Systems of orthogonal polynomials defined by hypergeometric type equations.

Abstract.
A hypergeometric type equation satisfying certain conditions defines either a finite or an infinite system of orthogonal polynomials. We present in a unified and explicit way all these systems of orthogonal polynomials, the associated special functions and the corresponding raising/lowering operators. This general formalism allows us to extend some known results to a larger class of functions.

Key Words.
orthogonal polynomials, associated special functions, raising operator, lowering operator, special functions

AMS(MOS) Subject Classifications.
33C45, 81R05, 81R30

55 *M. Isabel Bueno, Francisco Marcellán, and Jorge Sánchez-Ruiz.* Continuous symmetrized Sobolev inner products of order N (II).

Abstract.
Given a symmetrized Sobolev inner product of order N, the corresponding sequence of monic orthogonal polynomials $\{Q_n\}$ satisfies $Q_{2n}(x) = P_n(x^2)$, $Q_{2n+1}(x) = xR_n(x^2)$ for certain sequences of monic polynomials $\{P_n\}$ and $\{R_n\}$. In this paper we consider the particular case when all the measures that define the symmetrized Sobolev inner product are equal, absolutely continuous and semiclassical. Under

such restrictions, we give explicit algebraic relations between the sequences $\{P_n\}$ and $\{R_n\}$, as well as higher-order recurrence relations that they satisfy.

Key Words.
Sobolev inner product, orthogonal polynomials, semiclassical linear functionals, recurrence relation, symmetrization process

AMS(MOS) Subject Classifications.
42C05

66 Another approach to vibration analysis of stepped structures. *Igor Fedotov, Steve Joubert, Julian Marais, and Michael Shatalov*.

Abstract.
In this paper a model of an N–stepped bar with variable Cross-sections coupled with foundation by means of lumped masses and springs is studied. It is assumed that the process of vibrations in each section of the bar is described by a wave equation. The analytical tools of vibration analysis are based on finding eigenfunctions with piecewise continuous derivatives, which are orthogonal with respect to a generalized weight function. These eigenfunctions automatically satisfy the boundary conditions at the end points as well as the non-classical boundary conditions at the junctions. The solution of the problems is formulated in terms of Green function. By means of the proposed algorithm a problem of arbitrary complexity could be considered in the same terms as a single homogeneous bar. This algorithm is efficient in design of low frequency transducers. An example is given to show the practical application of the algorithm to a two-stepped transducer.

Key Words.
PDE with discontinuous coefficients, numerical approximation of eigenvalues, stepped structure, transducers, waveguide, variable cross-section, non-classical boundary conditions, Green function, resonance

AMS(MOS) Subject Classifications.
35B34, 35R05, 34B27, 34L16

74 q-orthogonal polynomials related to the quantum group $U_q(so(5))$. *Alexander Rozenblyum*.

Abstract.
Orthogonal polynomials in two discrete variables related to finite-dimensional irreducible representations of the quantum algebra $U_q(\mathfrak{so}(5))$ are studied. The polynomials we consider here can be treated as two-dimensional q-analogs of Krawtchouk polynomials. Some properties of these polynomials are investigated: the difference equation of the Sturm-Liouville type, the weight function, the corresponding eigenvalues including the explicit description of their multiplicities.

Key Words.
quantum group, discrete orthogonal polynomials, eigenvalues

AMS(MOS) Subject Classifications.
33D80, 33C45

79 Moment matrix of self-similar measures. *C. Escribano, M. A. Sastre, and E. Torrano*.

Abstract.
We give in this paper an expression for the moment matrix associated to a self-similar measure given by an Iterated Function Systems (IFS). This expression translates the self-similarity property of a measure to its moment matrix.
This matrix relation shows that the properties of a measure are reflected, not only in the equation of its Jacobi matrix, as stated in Krein theorem, but also in the moment matrix.

Key Words.
self-similar measures, orthogonal polynomials, moment matrix

AMS(MOS) Subject Classifications.
42C05, 28A80

88 Generalized weighted Sobolev spaces and applications to Sobolev orthogonal polynomials: a survey. *José M. Rodríguez, Venancio Álvarez, Elena Romera, and Domingo Pestana*.

Abstract.
In this paper we present a definition of Sobolev spaces with respect to general measures, prove some useful technical results, some of them generalizations of classical results with Lebesgue measure and find general conditions under which these spaces are complete. These results have important consequences in Approximation Theory. We also find conditions under which the evaluation operator is bounded.

Key Words.
Sobolev spaces, weights, orthogonal polynomials

AMS(MOS) Subject Classifications.
41A10, 46E35, 46G10

94 Noncommutative extensions of Ramanujan's ${}_1\psi_1$ summation. *Michael Schlosser*.

Abstract.
Using functional equations, we derive noncommutative extensions of Ramanujan's ${}_1\psi_1$ summation.

Key Words.
noncommutative basic hypergeometric series, Ramanujan's ${}_1\psi_1$ summation

AMS(MOS) Subject Classifications.
33D15, 33D99

103 Weierstrass' theorem in weighted Sobolev spaces with k derivatives: announcement of results. *Ana Portilla, Yamilet Quintana, José M. Rodríguez, and Eva Tourís*.

Abstract.
We characterize the set of functions which can be approximated by smooth functions and by polynomials with the norm

$$\|f\|_{W^{k,\infty}(w)} := \sum_{j=0}^{k} \|f^{(j)}\|_{L^\infty(w_j)},$$

for a wide range of (even non-bounded) weights w_j's. We allow a great deal of independence among the weights w_j's.

Key Words.
Weierstrass' theorem, weight, Sobolev spaces, weighted Sobolev spaces

AMS(MOS) Subject Classifications.
41A10, 46E35, 46G10

108 Duality of q-polynomials, orthogonal on countable sets of points. *N. M. Atakishiyev and U. Klimyk*.

Abstract.
We review properties of q-orthogonal polynomials, related to their orthogonality, duality and connection with the theory of symmetric (self-adjoint) operators, represented by a Jacobi matrix. In particular, we show how one can naturally interpret the duality of families of polynomials, orthogonal on countable sets of points. In order to obtain orthogonality relations for dual sets of polynomials, we propose to use two symmetric (self-adjoint) operators, representable (in some distinct bases) by Jacobi matrices. To illustrate applications of this approach, we apply it to several pairs of dual families of q-polynomials, orthogonal on countable sets, from the q-Askey scheme. For each such pair, the corresponding operators, representable by Jacobi matrices, are explicitly given. These operators are employed in order to find explicitly sets of points, on which the polynomials are orthogonal, and orthogonality relations for them.

Key Words.
q-orthogonal polynomials, duality, Jacobi matrix, orthogonality relations

AMS(MOS) Subject Classifications.
33D80; 33D45; 17B37