

GENERALIZED WEIGHTED SOBOLEV SPACES AND APPLICATIONS TO SOBOLEV ORTHOGONAL POLYNOMIALS: A SURVEY*

JOSÉ M. RODRÍGUEZ[†], VENANCIO ÁLVAREZ[‡], ELENA ROMERA[§], AND DOMINGO PESTANA[¶]

Abstract. In this paper we present a definition of Sobolev spaces with respect to general measures, prove some useful technical results, some of them generalizations of classical results with Lebesgue measure and find general conditions under which these spaces are complete. These results have important consequences in Approximation Theory. We also find conditions under which the evaluation operator is bounded.

Key words. Sobolev spaces, weights, orthogonal polynomials

AMS subject classifications. 41A10, 46E35, 46G10

*Received November 30, 2004. Accepted for publication February 28, 2005. Recommended by J. Arvesú.

[†]Dep. de Matemáticas, Univ. Carlos III de Madrid, Avda. de la Universidad, 30, 28911 Leganés (Madrid), SPAIN (jomaro@math.uc3m.es). Research partially supported by grants from DGI (BFM 2003-06335-C03-02 and BFM 2003-04870), Spain.

[‡]Dep. de Análisis Matemático, Facultad de Ciencias, Campus de Teatinos, 29071 Málaga, SPAIN (nancho@anamat.cie.uma.es). Research partially supported by grants from MCYT (MTM 2004-00078) and Junta de Andalucía (FQM-210), Spain.

[§]Dep. de Matemáticas, Univ. Carlos III de Madrid, Avda. de la Universidad, 30, 28911 Leganés (Madrid), SPAIN (eromera@math.uc3m.es). Research partially supported by a grant from DGI (BFM 2003-06335-C03-02), Spain.

[¶]Dep. de Matemáticas, Univ. Carlos III de Madrid, Avda. de la Universidad, 30, 28911 Leganés (Madrid), SPAIN (dompes@math.uc3m.es). Research partially supported by grants from DGI (BFM 2003-06335-C03-02 and BFM 2003-04870), Spain.