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Contents

of the real axis as in the isotropic case. We show rigorously why this occurs for a semi-infinite and bounded interval. We then extend the gridding algorithm to hyperbolic problems on bounded domains. For the propagative modes, the problem is reduced to a rational approximation on an interval of the negative real semiaxis, similarly to in the isotropic case. For the wave problem we present numerical examples in 2-D anisotropic media.

Key Words.
finite differences, DtN maps, anisotropy, spectral approximation

AMS(MOS) Subject Classifications.
65M06, 65N06

82 *Drahoslava Janovská and Gerhard Opfer*. Computing quaternionic roots by Newton's method.

Abstract.
Newton's method for finding zeros is formally adapted to finding roots of Hamilton's quaternions. Since a derivative in the sense of complex analysis does not exist for quaternion valued functions we compare the resulting formulas with the more classical formulas obtained by using the Jacobian matrix and the Gâteaux derivative. The latter case includes also the so-called damped Newton form. We investigate the convergence behavior and show that under one simple condition all cases introduced, produce the same iteration sequence and have thus the same convergence behavior, namely that of locally quadratic convergence. By introducing an analogue of Taylor's formula for $x^n,\ n \in \mathbb{Z}$, we can show the local, quadratic convergence independently of the general theory. It will also be shown that the application of damping proves to be very useful. By applying Newton iterations backwards we detect all points for which the iteration (after a finite number of steps) must terminate. These points form a nice pattern. There are explicit formulas for roots of quaternions and also numerical examples.

Key Words.
roots of quaternions, Newton's method applied to finding roots of quaternions

AMS(MOS) Subject Classifications.
11R52, 12E15, 30G35, 65D15

103 A FETI-DP preconditioner for mortar methods in three dimensions. *Hyea Hyun Kim*.

Abstract.
A FETI-DP method is developed for three dimensional elliptic problems with mortar discretization. Mortar matching conditions are considered as the continuity constraints in the FETI-DP formulation. Among them, face average constraints are selected as primal constraints in our FETI-DP formulation to achieve an algorithm as scalable as two dimensional problems. A Neumann-Dirichlet preconditioner is used in the FETI-DP formulation and it gives the condition number bound

$$C \max_{i=1,\cdots,N} \left\{ \left(1 + \log\left(H_i/h_i\right)\right)^2 \right\},$$

where H_i and h_i are sizes of domain and mesh for each subdomain, respectively. When the subdomain with the smaller coefficient is chosen as the nonmortar side

across the interface, the constant C is independent of H_i, h_i, and the coefficients of the elliptic problem. The proposed algorithm can be applied to two dimensional elliptic problems with edge average constraints only as primal constraints and it can be generalized to geometrically non-conforming subdomain partitions. Numerical results present the performance of the algorithm for elliptic problems with discontinuous coefficients.

Key Words.
FETI-DP, non-matching grids, mortar methods, preconditioner

AMS(MOS) Subject Classifications.
65N30, 65N55

121 The parametrized SR algorithm for Hamiltonian matrices. *H. Faßbender*.

Abstract.
The heart of the implicitly restarted symplectic Lanczos method for Hamiltonian matrices consists of the SR algorithm, a structure-preserving algorithm for computing the spectrum of Hamiltonian matrices. The symplectic Lanczos method projects the large, sparse $2n \times 2n$ Hamiltonian matrix H onto a small, dense $2k \times 2k$ Hamiltonian J-Hessenberg matrix $\widetilde{H}$, $k \ll n$. This $2k \times 2k$ Hamiltonian matrix is uniquely determined by $4k - 1$ parameters. Using these $4k - 1$ parameters, one step of the SR algorithm can be carried out in $\mathcal{O}(k)$ arithmetic operations (compared to $\mathcal{O}(k^3)$ arithmetic operations when working on the actual Hamiltonian matrix). As in the context of the implicitly restarted symplectic Lanczos method the usual assumption, that the Hamiltonian eigenproblem to be solved is stable, does not hold, the case of purely imaginary eigenvalues in the SR algorithm is treated here.

Key Words.
Hamiltonian matrix, eigenvalue problem, SR algorithm

AMS(MOS) Subject Classifications.
65F15

146 A BDDC algorithm for flow in porous media with a hybrid finite element discretization. *Xuemin Tu*.

Abstract.
The BDDC (balancing domain decomposition by constraints) methods have been applied successfully to solve the large sparse linear algebraic systems arising from conforming finite element discretizations of elliptic boundary value problems. In this paper, the scalar elliptic problems for flow in porous media are discretized by a hybrid finite element method which is equivalent to a nonconforming finite element method. The BDDC algorithm is extended to these problems which originate as saddle point problems. Edge/face average constraints are enforced across the interface and the same rate of convergence is obtained as in conforming cases. The condition number of the preconditioned system is estimated and numerical experiments are discussed.

Key Words.
BDDC, domain decomposition, saddle point problem, condition number, hybrid finite element method

AMS(MOS) Subject Classifications.
65N30, 65N55, 65F10

Abstract.
In this paper we introduce a new method for the computation of KKT matrices that arise from solving constrained, nonlinear optimization problems. This method requires updating of null-space factorizations after a low rank modification. The update procedure has the advantage that it is significantly cheaper than a re-factorization of the system at each new iterate. This paper focuses on the cheap update of a rectangular LU-decomposition after a rank-1 modification. Two different procedures for updating the LU-factorization are presented in detail and compared regarding their costs of computation and their stability. Moreover we will introduce an extension of these algorithms which further improves the computation time. This turns out to be an excellent alternative to algorithms based on orthogonal transformations.

Key Words.
KKT-system, LU-factorization, low-rank modification, quasi-Newton method

AMS(MOS) Subject Classifications.
15A23, 65F05, 65F30, 65K05, 90C53

Abstract.
The difficult factor in the condition number $\|A\|\,\|A^{-1}\|$ of a large linear system $Ap = y$ is the spectral norm of A^{-1}. To eliminate this factor, we here replace worst case analysis by a probabilistic argument. To be more precise, we randomly take p from a ball with the uniform distribution and show that then, with a certain probability close to one, the relative errors $\|\Delta p\|$ and $\|\Delta y\|$ satisfy $\|\Delta p\| \le C\|\Delta y\|$ with a constant C that involves only the Frobenius and spectral norms of A. The success of this argument is demonstrated for Toeplitz systems and for the problem of sampling multivariate trigonometric polynomials on nonuniform knots. The limitations of the argument are also shown.

Key Words.
condition number, probability argument, linear system, Toeplitz matrix, nonuniform sampling, multivariate trigonometric polynomial

AMS(MOS) Subject Classifications.
65F35, 15A12, 47B35, 60H25, 94A20

Abstract.
We present second order extensions of the Hilber-Hughes-Taylor-α (HHT-α) method for systems of overdetermined differential-algebraic equations (ODAEs) arising, for example, in mechanics. A detailed analysis of extensions of the HHT-α method is given. In particular a local and global error analysis is presented. Second

will derive a solver of complexity $O(p_1 p_2 n)$. The system solver is written inductively, and uses in every step k, the solution of a so-called kth order Yule-Walker-like equation. The algorithm obtained first has complexity $O(p_1 p_2 n^2)$. Based, however on the specific structure of the simple (p_1, p_2)-Levinson conform matrices, we will be able to further reduce the complexity of the presented method, and get an order $O(p_1 p_2 n)$ algorithm.
Different examples of matrices are given for this algorithm. Examples are presented for: general dense matrices, upper triangular matrices, higher order generator semiseparable matrices, quasiseparable matrices, Givens-vector representable semiseparable matrices, band matrices, companion matrices, confederate matrices, arrowhead matrices, fellow matrices and many more.
Finally, the relation between this method and an upper triangular factorization of the original matrix is given and also details concerning possible look ahead methods are presented.

Key Words.
Levinson, Yule-Walker, look-ahead, system solving, Levinson conform matrices

AMS(MOS) Subject Classifications.
65F05

270 Analysis of the $\boldsymbol{A}, V - \boldsymbol{A} - \psi$ potential formulation for the eddy current problem in a bounded domain. *Ramiro Acevedo and Rodolfo Rodríguez*.

Abstract.
The aim of this paper is to provide a mathematical analysis of the well-known $\boldsymbol{A}, V - \boldsymbol{A} - \psi$ potential formulation for the eddy current problem. The resulting variational problem is proved to be well posed and error estimates are settled for a numerical method based on standard nodal finite elements.

Key Words.
eddy currents, potential formulation, well-posedness, finite elements, error estimates

AMS(MOS) Subject Classifications.
78M10, 65N30

285 Joint domain-decomposition $\mathcal{H}$-LU preconditioners for saddle point problems. *Sabine Le Borne and Suely Oliveira*.

Abstract.
For saddle point problems in fluid dynamics, several popular preconditioners exploit the block structure of the problem to construct block triangular preconditioners. The performance of such preconditioners depends on whether fast, approximate solvers for the linear systems on the block diagonal (representing convection-diffusion problems) as well as for the Schur complement (in the pressure variables) are available. In this paper, we will introduce a completely different approach in which we ignore this given block structure. We will instead compute an approximate LU-factorization of the complete system matrix using hierarchical matrix techniques. In particular, we will use domain-decomposition clustering with an additional local pivoting strat-

coefficients with respect to Jacobi orthogonal polynomials. Then we give explicitly the optimal constrained one degree reduction in Bézier form, by perturbing the Bézier coefficients.

Key Words.
reduction, polynomials, approximation, Bézier curves

AMS(MOS) Subject Classifications.
41A10, 65D05, 65D17

320 *Natacha Fontes, Janice Kover, Laura Smithies, and Richard S. Varga*. Singular value decomposition normally estimated Geršgorin sets.

Abstract.
Let $B \in \mathbb{C}^{N\times N}$ denote a finite-dimensional square complex matrix, and let $V\Sigma W^*$ denote a fixed singular value decomposition (SVD) of B. In this note, we follow up work from Smithies and Varga [Linear Algebra Appl., 417 (2006), pp. 370–380], by defining the SV-normal estimator $\epsilon_{V\Sigma W^*}$, (which satisfies $0 \leq \epsilon_{V\Sigma W^*} \leq 1$), and showing how it defines an upper bound on the norm, $\|B^*B - BB^*\|_2$, of the commutant of B and its adjoint, $B^* = \bar{B}^T$. We also introduce the SV-normally estimated Geršgorin set, $\Gamma^{\rm NSV}(V\Sigma W^*)$, of B, defined by this SVD. Like the Geršgorin set for B, the set $\Gamma^{\rm NSV}(V\Sigma W^*)$ is a union of N closed discs which contains the eigenvalues of B. When $\epsilon_{V\Sigma W^*}$ is zero, $\Gamma^{\rm NSV}(V\Sigma W^*)$ is exactly the set of eigenvalues of B; when $\epsilon_{V\Sigma W^*}$ is small, the set $\Gamma^{\rm NSV}(V\Sigma W^*)$ provides a good estimate of the spectrum of B. We end this note by expanding on an example from Smithies and Varga [Linear Algebra Appl., 417 (2006), pp. 370–380], and giving some examples which were generated using Matlab of the sets $\Gamma^{\rm NSV}(V\Sigma W^*)$ and $\Gamma^{\rm RNSV}(V\Sigma W^*)$, the reduced SV-normally estimated Geršgorin set.

Key Words.
Geršgorin type sets, normal matrices, eigenvalue estimates

AMS(MOS) Subject Classifications.
15A18, 47A07

330 *J. M. Tang and C. Vuik*. Efficient deflation methods applied to 3-D bubbly flow problems.

Abstract.
For various applications, it is well-known that deflated ICCG is an efficient method to solve linear systems with an invertible coefficient matrix. Tang and Vuik [J. Comput. Appl. Math., 206 (2007), pp. 603–614] proposed two equivalent variants of this deflated method, which can also solve linear systems with singular coefficient matrices that arise from the discretization of the Poisson equation with Neumann boundary conditions and discontinuous coefficients. In this paper, we also consider the original variant of DICCG in Vuik, Segal, and Meijerink [J. Comput. Phys., 152 (1999), pp. 385–403], that already proved its efficiency for invertible coefficient matrices. This variant appears to be theoretically equivalent to the first two variants, so that they all have the same convergence properties. Moreover, we show that the associated coarse linear systems within these variants can be solved both directly and iteratively. In applications with large grid sizes, the method with the iterative coarse solver can be substantially more efficient than the one with the standard direct coarse solver.

residual norm) in case of a normal matrix A, but it need not characterize the worst-case GMRES behavior if A is nonnormal. Characterizing the tightness of this bound for nonnormal matrices A represents an important and largely open problem in the convergence analysis of Krylov subspace methods. In this paper we address this problem in case A is a single Jordan block. We study the relation between ideal and worst-case GMRES as well as the problem of estimating the ideal GMRES approximation. Furthermore, we prove new results about the radii of the polynomial numerical hulls of Jordan blocks. Using these, we discuss the closeness of the lower bound on the ideal GMRES approximation that is derived from the radius of the polynomial numerical hull.

Key Words.
GMRES convergence, ideal GMRES, polynomial numerical hull, Jordan block.

AMS(MOS) Subject Classifications.
65F10, 65F35, 49K35.

474 *Neville J. Ford and Patricia M. Lumb*. Theory and numerics for multi-term periodic delay differential equations: small solutions and their detection.

Abstract.
In this paper we consider scalar linear periodic delay differential equations of the form

$$x'(t) = \sum_{j=0}^{m} b_j(t)x(t-jw), x(t) = \phi(t) \text{ for } t \in [0, mw),\ t \geq mw \quad (\ddagger)$$

where b_j, $j = 0, ..., m$ are continuous periodic functions with period w. We summarise a theoretical treatment that analyses whether the equation has small solutions. We consider discrete equations that arise when a numerical method with fixed step-size is applied to approximate the solution to (‡) and we develop a corresponding theory. Our results show that small solutions can be detected reliably by the numerical scheme. We conclude with some numerical examples.

Key Words.
delay differential equations, small solutions, super-exponential solutions, numerical methods

AMS(MOS) Subject Classifications.
34K28, 65P99, 37N30