Electronic Transactions on Numerical Analysis
Volume 26, 2007

Contents

of the real axis as in the isotropic case. We show rigorously why this occurs for a semi-infinite and bounded interval. We then extend the gridding algorithm to hyperbolic problems on bounded domains. For the propagative modes, the problem is reduced to a rational approximation on an interval of the negative real semiaxis, similarly to in the isotropic case. For the wave problem we present numerical examples in 2-D anisotropic media.

Key Words.
finite differences, DtN maps, anisotropy, spectral approximation

AMS(MOS) Subject Classifications.
65M06, 65N06

82 *Drahoslava Janovská and Gerhard Opfer*. Computing quaternionic roots by Newton's method.

Abstract.
Newton's method for finding zeros is formally adapted to finding roots of Hamilton's quaternions. Since a derivative in the sense of complex analysis does not exist for quaternion valued functions we compare the resulting formulas with the more classical formulas obtained by using the Jacobian matrix and the Gâteaux derivative. The latter case includes also the so-called damped Newton form. We investigate the convergence behavior and show that under one simple condition all cases introduced, produce the same iteration sequence and have thus the same convergence behavior, namely that of locally quadratic convergence. By introducing an analogue of Taylor's formula for $x^n,\ n \in \mathbb{Z}$, we can show the local, quadratic convergence independently of the general theory. It will also be shown that the application of damping proves to be very useful. By applying Newton iterations backwards we detect all points for which the iteration (after a finite number of steps) must terminate. These points form a nice pattern. There are explicit formulas for roots of quaternions and also numerical examples.

Key Words.
roots of quaternions, Newton's method applied to finding roots of quaternions

AMS(MOS) Subject Classifications.
11R52, 12E15, 30G35, 65D15

103 A FETI-DP preconditioner for mortar methods in three dimensions. *Hyea Hyun Kim*.

Abstract.
A FETI-DP method is developed for three dimensional elliptic problems with mortar discretization. Mortar matching conditions are considered as the continuity constraints in the FETI-DP formulation. Among them, face average constraints are selected as primal constraints in our FETI-DP formulation to achieve an algorithm as scalable as two dimensional problems. A Neumann-Dirichlet preconditioner is used in the FETI-DP formulation and it gives the condition number bound

$$C \max_{i=1,\cdots,N} \left\{ (1+\log(H_i/h_i))^2 \right\},$$

where H_i and h_i are sizes of domain and mesh for each subdomain, respectively. When the subdomain with the smaller coefficient is chosen as the nonmortar side across the interface, the constant C is independent of H_i, h_i, and the coefficients of the elliptic problem. The proposed algorithm can be applied to two dimensional elliptic problems with edge average constraints only as primal constraints and it can be generalized to geometrically non-conforming subdomain partitions. Numerical results present the performance of the algorithm for elliptic problems with discontinuous coefficients.

Key Words.
FETI-DP, non-matching grids, mortar methods, preconditioner

AMS(MOS) Subject Classifications.
65N30, 65N55

Abstract.
The heart of the implicitly restarted symplectic Lanczos method for Hamiltonian matrices consists of the SR algorithm, a structure-preserving algorithm for computing the spectrum of Hamiltonian matrices. The symplectic Lanczos method projects the large, sparse $2n \times 2n$ Hamiltonian matrix H onto a small, dense $2k \times 2k$ Hamiltonian J-Hessenberg matrix $\widetilde{H}$, $k \ll n$. This $2k \times 2k$ Hamiltonian matrix is uniquely determined by $4k - 1$ parameters. Using these $4k - 1$ parameters, one step of the SR algorithm can be carried out in $\mathcal{O}(k)$ arithmetic operations (compared to $\mathcal{O}(k^3)$ arithmetic operations when working on the actual Hamiltonian matrix). As in the context of the implicitly restarted symplectic Lanczos method the usual assumption, that the Hamiltonian eigenproblem to be solved is stable, does not hold, the case of purely imaginary eigenvalues in the SR algorithm is treated here.

Key Words.
Hamiltonian matrix, eigenvalue problem, SR algorithm

AMS(MOS) Subject Classifications.
65F15

Abstract.
The BDDC (balancing domain decomposition by constraints) methods have been applied successfully to solve the large sparse linear algebraic systems arising from conforming finite element discretizations of elliptic boundary value problems. In this paper, the scalar elliptic problems for flow in porous media are discretized by a hybrid finite element method which is equivalent to a nonconforming finite element method. The BDDC algorithm is extended to these problems which originate as saddle point problems. Edge/face average constraints are enforced across the interface and the same rate of convergence is obtained as in conforming cases. The condition number of the preconditioned system is estimated and numerical experiments are discussed.

Key Words.
BDDC, domain decomposition, saddle point problem, condition number, hybrid finite element method

AMS(MOS) Subject Classifications.
65N30, 65N55, 65F10

161 On the efficient update of rectangular LU-factorizations subject to low rank modifications. *Peter Stange, Andreas Griewank, and Matthias Bollhöfer.*

Abstract.
In this paper we introduce a new method for the computation of KKT matrices that arise from solving constrained, nonlinear optimization problems. This method requires updating of null-space factorizations after a low rank modification. The update procedure has the advantage that it is significantly cheaper than a re-factorization of the system at each new iterate. This paper focuses on the cheap update of a rectangular LU-decomposition after a rank-1 modification. Two different procedures for updating the LU-factorization are presented in detail and compared regarding their costs of computation and their stability. Moreover we will introduce an extension of these algorithms which further improves the computation time. This turns out to be an excellent alternative to algorithms based on orthogonal transformations.

Key Words.
KKT-system, LU-factorization, low-rank modification, quasi-Newton method

AMS(MOS) Subject Classifications.
15A23, 65F05, 65F30, 65K05, 90C53

178 Probability against condition number and sampling of multivariate trigonometric random polynomials. *Albrecht Böttcher and Daniel Potts.*

Abstract.
The difficult factor in the condition number $\|A\|\,\|A^{-1}\|$ of a large linear system $Ap = y$ is the spectral norm of A^{-1}. To eliminate this factor, we here replace worst case analysis by a probabilistic argument. To be more precise, we randomly take p from a ball with the uniform distribution and show that then, with a certain probability close to one, the relative errors $\|\Delta p\|$ and $\|\Delta y\|$ satisfy $\|\Delta p\| \leq C\|\Delta y\|$ with a constant C that involves only the Frobenius and spectral norms of A. The success of this argument is demonstrated for Toeplitz systems and for the problem of sampling multivariate trigonometric polynomials on nonuniform knots. The limitations of the argument are also shown.

Key Words.
condition number, probability argument, linear system, Toeplitz matrix, nonuniform sampling, multivariate trigonometric polynomial

AMS(MOS) Subject Classifications.
65F35, 15A12, 47B35, 60H25, 94A20

190 Extensions of the HHT-α method to differential-algebraic equations in mechanics. *Laurent O. Jay and Dan Negrut.*

Abstract.
We present second order extensions of the Hilber-Hughes-Taylor-α (HHT-α) method for systems of overdetermined differential-algebraic equations (ODAEs) arising, for example, in mechanics. A detailed analysis of extensions of the HHT-α method is given. In particular a local and global error analysis is presented. Second

order convergence is theoretically demonstrated and practically illustrated by numerical experiments. A new variable stepsize formula is proposed which preserves the second order of the method.

Key Words.
differential-algebraic equations, HHT-α method, variable stepsize

AMS(MOS) Subject Classifications.
65L05, 65L06, 65L80, 70F20, 70H45

209 Block triangular preconditioners for M-matrices and Markov chains. *Michele Benzi and Bora Uçar*.

Abstract.
We consider preconditioned Krylov subspace methods for solving large sparse linear systems under the assumption that the coefficient matrix is a (possibly singular) M-matrix. The matrices are partitioned into 2×2 block form using graph partitioning. Approximations to the Schur complement are used to produce various preconditioners of block triangular and block diagonal type. A few properties of the preconditioners are established, and extensive numerical experiments are used to illustrate the performance of the various preconditioners on singular linear systems arising from Markov modeling.

Key Words.
M-matrices, preconditioning, discrete Markov chains, iterative methods, graph partitioning

AMS(MOS) Subject Classifications.
05C50, 60J10, 60J22, 65F10, 65F35, 65F50

228 Piecewise bilinear preconditioning of high-order finite element method. *Sang Dong Kim*.

Abstract.
Bounds on eigenvalues which are independent of both degrees of high-order elements and mesh sizes are shown for the system preconditioned by bilinear elements for high-order finite element discretizations applied to a model uniformly elliptic operator.

Key Words.
multigrid, high-order finite element methods, piecewise linear preconditioning

AMS(MOS) Subject Classifications.
65F10, 65M30

243 Solving linear systems with a Levinson-like solver. *Raf Vandebril, Nicola Mastronardi, and Marc Van Barel*.

Abstract.
In this paper we will present a general framework for solving linear systems of equations. The solver is based on the Levinson-idea for solving Toeplitz systems of equations. We will consider a general class of matrices, defined as the class of simple (p_1, p_2)-Levinson conform matrices. This class incorporates, for instance, semiseparable, band, companion, arrowhead and many other matrices. For this class, we

will derive a solver of complexity $O(p_1 p_2 n)$. The system solver is written inductively, and uses in every step k, the solution of a so-called kth order Yule-Walker-like equation. The algorithm obtained first has complexity $O(p_1 p_2 n^2)$. Based, however on the specific structure of the simple (p_1, p_2)-Levinson conform matrices, we will be able to further reduce the complexity of the presented method, and get an order $O(p_1 p_2 n)$ algorithm.
Different examples of matrices are given for this algorithm. Examples are presented for: general dense matrices, upper triangular matrices, higher order generator semiseparable matrices, quasiseparable matrices, Givens-vector representable semiseparable matrices, band matrices, companion matrices, confederate matrices, arrowhead matrices, fellow matrices and many more.
Finally, the relation between this method and an upper triangular factorization of the original matrix is given and also details concerning possible look ahead methods are presented.

Key Words.
Levinson, Yule-Walker, look-ahead, system solving, Levinson conform matrices

AMS(MOS) Subject Classifications.
65F05

270 Analysis of the $\boldsymbol{A}, V - \boldsymbol{A} - \psi$ potential formulation for the eddy current problem in a bounded domain. *Ramiro Acevedo and Rodolfo Rodríguez*.

Abstract.
The aim of this paper is to provide a mathematical analysis of the well-known $\boldsymbol{A}, V - \boldsymbol{A} - \psi$ potential formulation for the eddy current problem. The resulting variational problem is proved to be well posed and error estimates are settled for a numerical method based on standard nodal finite elements.

Key Words.
eddy currents, potential formulation, well-posedness, finite elements, error estimates

AMS(MOS) Subject Classifications.
78M10, 65N30

285 Joint domain-decomposition $\mathcal{H}$-LU preconditioners for saddle point problems. *Sabine Le Borne and Suely Oliveira*.

Abstract.
For saddle point problems in fluid dynamics, several popular preconditioners exploit the block structure of the problem to construct block triangular preconditioners. The performance of such preconditioners depends on whether fast, approximate solvers for the linear systems on the block diagonal (representing convection-diffusion problems) as well as for the Schur complement (in the pressure variables) are available. In this paper, we will introduce a completely different approach in which we ignore this given block structure. We will instead compute an approximate LU-factorization of the complete system matrix using hierarchical matrix techniques. In particular, we will use domain-decomposition clustering with an additional local pivoting strat-

egy to order the complete index set. As a result, we obtain an $\mathcal{H}$-matrix structure in which an $\mathcal{H}$-LU factorization is computed more efficiently and with higher accuracy than for the corresponding block structure based clustering. $\mathcal{H}$-LU preconditioners resulting from the block and joint approaches will be discussed and compared through numerical results.

Key Words.
hierarchical matrices, data-sparse approximation, Oseen equations, preconditioning, factorization

AMS(MOS) Subject Classifications.
65F05, 65F30, 65F50

299 Iterative methods for solving the dual formulation arising from image restoration. *Tony F. Chan, Ke Chen, and Jamylle L. Carter*.

Abstract.
Many variational models for image denoising restoration are formulated in primal variables that are directly linked to the solution to be restored. If the total variation related semi-norm is used in the models, one consequence is that extra regularization is needed to remedy the highly non-smooth and oscillatory coefficients for effective numerical solution. The dual formulation was often used to study theoretical properties of a primal formulation. However as a model, this formulation also offers some advantages over the primal formulation in dealing with the above mentioned oscillation and non-smoothness. This paper presents some preliminary work on speeding up the Chambolle method [J. Math. Imaging Vision, 20 (2004), pp. 89–97] for solving the dual formulation. Following a convergence rate analysis of this method, we first show why the nonlinear multigrid method encounters some difficulties in achieving convergence. Then we propose a modified smoother for the multigrid method to enable it to achieve convergence in solving a regularized Chambolle formulation. Finally, we propose a linearized primal-dual iterative method as an alternative stand-alone approach to solve the dual formulation without regularization. Numerical results are presented to show that the proposed methods are much faster than the Chambolle method.

Key Words.
image restoration, nonlinear partial differential equations, singularity, nonlinear iterations, Fourier analysis, multigrid method

AMS(MOS) Subject Classifications.
68U10, 65F10, 65K10

312 *Germain E. Randriambelosoa*. Polynomial best constrained degree reduction in strain energy.

Abstract.
We exhibit the best degree reduction of a given degree n polynomial by minimizing the strain energy of the error with the constraint that continuity of a prescribed order is preserved at the two endpoints. It is shown that a multidegree reduction is equivalent to a step-by-step reduction of one degree at a time by using the Fourier

coefficients with respect to Jacobi orthogonal polynomials. Then we give explicitly the optimal constrained one degree reduction in Bézier form, by perturbing the Bézier coefficients.

Key Words.
reduction, polynomials, approximation, Bézier curves

AMS(MOS) Subject Classifications.
41A10, 65D05, 65D17

320 *Natacha Fontes, Janice Kover, Laura Smithies, and Richard S. Varga*. Singular value decomposition normally estimated Geršgorin sets.

Abstract.
Let $B \in \mathbb{C}^{N \times N}$ denote a finite-dimensional square complex matrix, and let $V\Sigma W^*$ denote a fixed singular value decomposition (SVD) of B. In this note, we follow up work from Smithies and Varga [Linear Algebra Appl., 417 (2006), pp. 370–380], by defining the SV-normal estimator $\epsilon_{V\Sigma W^*}$, (which satisfies $0 \leq \epsilon_{V\Sigma W^*} \leq 1$), and showing how it defines an upper bound on the norm, $\|B^*B - BB^*\|_2$, of the commutant of B and its adjoint, $B^* = \bar{B}^T$. We also introduce the SV-normally estimated Geršgorin set, $\Gamma^{\mathrm{NSV}}(V\Sigma W^*)$, of B, defined by this SVD. Like the Geršgorin set for B, the set $\Gamma^{\mathrm{NSV}}(V\Sigma W^*)$ is a union of N closed discs which contains the eigenvalues of B. When $\epsilon_{V\Sigma W^*}$ is zero, $\Gamma^{\mathrm{NSV}}(V\Sigma W^*)$ is exactly the set of eigenvalues of B; when $\epsilon_{V\Sigma W^*}$ is small, the set $\Gamma^{\mathrm{NSV}}(V\Sigma W^*)$ provides a good estimate of the spectrum of B. We end this note by expanding on an example from Smithies and Varga [Linear Algebra Appl., 417 (2006), pp. 370–380], and giving some examples which were generated using Matlab of the sets $\Gamma^{\mathrm{NSV}}(V\Sigma W^*)$ and $\Gamma^{\mathrm{RNSV}}(V\Sigma W^*)$, the reduced SV-normally estimated Geršgorin set.

Key Words.
Geršgorin type sets, normal matrices, eigenvalue estimates

AMS(MOS) Subject Classifications.
15A18, 47A07

330 *J. M. Tang and C. Vuik*. Efficient deflation methods applied to 3-D bubbly flow problems.

Abstract.
For various applications, it is well-known that deflated ICCG is an efficient method to solve linear systems with an invertible coefficient matrix. Tang and Vuik [J. Comput. Appl. Math., 206 (2007), pp. 603–614] proposed two equivalent variants of this deflated method, which can also solve linear systems with singular coefficient matrices that arise from the discretization of the Poisson equation with Neumann boundary conditions and discontinuous coefficients. In this paper, we also consider the original variant of DICCG in Vuik, Segal, and Meijerink [J. Comput. Phys., 152 (1999), pp. 385–403], that already proved its efficiency for invertible coefficient matrices. This variant appears to be theoretically equivalent to the first two variants, so that they all have the same convergence properties. Moreover, we show that the associated coarse linear systems within these variants can be solved both directly and iteratively. In applications with large grid sizes, the method with the iterative coarse solver can be substantially more efficient than the one with the standard direct coarse solver.

Additionally, the results for stationary numerical experiments of Tang and Vuik [J. Comput. Appl. Math., 206 (2007), pp. 603–614] have only been given in terms of number of iterations. After discussing some implementation issues, we show in this paper that deflated ICCG is considerably faster than ICCG in the most test cases, by taking the computational time into account as well. Other 3-D time-dependent numerical experiments with falling droplets in air and rising air bubbles in water are performed, in order to show that deflated ICCG is also more efficient than ICCG in these cases, considering both the number of iterations and computational time.

Key Words.
deflation, conjugate gradient method, preconditioning, Poisson equation, symmetric positive semi-definite matrices, bubbly flow problems, inner-outer iterations

AMS(MOS) Subject Classifications.
65F10, 65F50, 65N22

350 *Juan Galvis and Marcus Sarkis*. Non-matching mortar discretization analysis for the coupling Stokes-Darcy equations.

Abstract.
We consider the coupling across an interface of fluid and porous media flows with Beavers-Joseph-Saffman transmission conditions. Under an adequate choice of Lagrange multipliers on the interface we analyze inf-sup conditions and optimal a priori error estimates associated with the continuous and discrete formulations of this Stokes-Darcy system. We allow the meshes of the two regions to be non-matching across the interface. Using mortar finite element analysis and appropriate scaled norms we show that the constants that appear on the a priori error bounds do not depend on the viscosity, permeability and ratio of mesh parameters. Numerical experiments are presented.

Key Words.
inf-sup condition, error estimates, mortar finite elements, multiphysics, porous media flow, incompressible fluid flow, Lagrange multipliers, saddle point problems, non-matching grids, discontinuous coefficients

AMS(MOS) Subject Classifications.
65N30, 65N15, 65N12, 35Q30, 35Q35, 76D033, 76D07

385 *Peter Kunkel and Volker Mehrmann*. Stability properties of differential-algebraic equations and spin-stabilized discretizations.

Abstract.
Classical stability properties of solutions that are well-known for ordinary differential equations (ODEs) are generalized to differential-algebraic equations (DAEs). A new test equation is derived for the analysis of numerical methods applied to DAEs with respect to the stability of the numerical approximations. Morevover, a stabilization technique is developed to improve the stability of classical DAE integration methods. The stability regions for these stabilized discretization methods are determined and it is shown that they much better reproduce the stability properties known

for the ODE case than in the unstabilized form. Movies that depict the stability regions for several methods are included for interactive use.

Key Words.
nonlinear differential-algebraic equations, stability, asymptotic stability, Lyapunov stability, spin-stabilized discretization, test equation, strangeness index

AMS(MOS) Subject Classifications.
65L80, 65L20, 34D20, 34D23

421 *Gabriel N. Gatica*. An augmented mixed finite element method for linear elasticity with non-homogeneous Dirichlet conditions.

Abstract.
We have recently developed a new augmented mixed finite element method for plane linear elasticity, which is based on the introduction of suitable Galerkin least-squares type terms. The corresponding analysis makes use of the first Korn inequality, and hence only null Dirichlet conditions, either on the whole boundary or on part of it, are considered. In the present paper we extend these results to the case of non-homogeneous Dirichlet boundary conditions. To this end, we incorporate additional consistent terms and then apply a slight extension of the classical Korn inequality. We show that the resulting augmented formulation and the associated Galerkin scheme are well posed. Finally, several numerical examples illustrating the good performance of the method are provided.

Key Words.
mixed-FEM, augmented formulation, linear elasticity

AMS(MOS) Subject Classifications.
65N30, 65N12, 65N15, 74B05

439 *Joris Van Deun*. Electrostatics and ghost poles in near best fixed pole rational interpolation.

Abstract.
We consider points that are near best for rational interpolation with prescribed poles in the same sense that Chebyshev points are near best for polynomial interpolation. It is shown that these interpolation points satisfy an electrostatic equilibrium problem involving the fixed poles and certain ‘ghost’ poles. This problem is closely related to Lamé equations with residues of mixed sign.

Key Words.
rational interpolation, Chebyshev weight, zeros, potential theory.

AMS(MOS) Subject Classifications.
Primary 33C45, secondary 42C05.

453 *Petr Tichý, Jörg Liesen, and Vance Faber*. On worst-case GMRES, ideal GMRES, and the polynomial numerical hull of a Jordan block.

Abstract.
When solving a linear algebraic system $Ax = b$ with GMRES, the relative residual norm at each step is bounded from above by the so-called ideal GMRES approximation. This worst-case bound is sharp (i.e. it is attainable by the relative GMRES

residual norm) in case of a normal matrix A, but it need not characterize the worst-case GMRES behavior if A is nonnormal. Characterizing the tightness of this bound for nonnormal matrices A represents an important and largely open problem in the convergence analysis of Krylov subspace methods. In this paper we address this problem in case A is a single Jordan block. We study the relation between ideal and worst-case GMRES as well as the problem of estimating the ideal GMRES approximation. Furthermore, we prove new results about the radii of the polynomial numerical hulls of Jordan blocks. Using these, we discuss the closeness of the lower bound on the ideal GMRES approximation that is derived from the radius of the polynomial numerical hull.

Key Words.
GMRES convergence, ideal GMRES, polynomial numerical hull, Jordan block.

AMS(MOS) Subject Classifications.
65F10, 65F35, 49K35.

474 *Neville J. Ford and Patricia M. Lumb*. Theory and numerics for multi-term periodic delay differential equations: small solutions and their detection.

Abstract.
In this paper we consider scalar linear periodic delay differential equations of the form

$$x'(t) = \sum_{j=0}^{m} b_j(t)x(t-jw), x(t) = \phi(t) \text{ for } t \in [0, mw),\ t \geq mw \quad (\ddagger)$$

where b_j, $j = 0, ..., m$ are continuous periodic functions with period w. We summarise a theoretical treatment that analyses whether the equation has small solutions. We consider discrete equations that arise when a numerical method with fixed step-size is applied to approximate the solution to (‡) and we develop a corresponding theory. Our results show that small solutions can be detected reliably by the numerical scheme. We conclude with some numerical examples.

Key Words.
delay differential equations, small solutions, super-exponential solutions, numerical methods

AMS(MOS) Subject Classifications.
34K28, 65P99, 37N30