BLOCK TRIANGULAR PRECONDITIONERS FOR $M$-MATRICES AND MARKOV CHAINS

MICHELE BENZI$^1$ AND BORA ÜÇAR$^2$

Abstract. We consider preconditioned Krylov subspace methods for solving large sparse linear systems under the assumption that the coefficient matrix is a (possibly singular) $M$-matrix. The matrices are partitioned into $2 \times 2$ block form using graph partitioning. Approximations to the Schur complement are used to produce various preconditioners of block triangular and block diagonal type. A few properties of the preconditioners are established, and extensive numerical experiments are used to illustrate the performance of the various preconditioners on singular linear systems arising from Markov modeling.

Key words. $M$-matrices, preconditioning, discrete Markov chains, iterative methods, graph partitioning

AMS subject classifications. 05C50, 60J10, 60J22, 65F10, 65F35, 65F50

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$^1$Department of Mathematics and Computer Science, Emory University, Atlanta, GA 30322, USA (benzi@mathcs.emory.edu). The work of this author was supported in part by the National Science Foundation grant DMS-0511336.

$^2$Department of Mathematics and Computer Science, Emory University, Atlanta, GA 30322, USA. Current Address: CERFACS, 42 Avenue G. Coriolis, 31057 Toulouse Cedex, France (ubora@cerfacs.fr). The work of this author was partially supported by The Scientific and Technological Research Council of Turkey (TUBITAK) and by the University Research Committee of Emory University.