SOLVING LINEAR SYSTEMS WITH A LEVINSON-LIKE SOLVER[*]

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Abstract. In this paper we will present a general framework for solving linear systems of equations. The solver is based on the Levinson-idea for solving Toeplitz systems of equations. We will consider a general class of matrices, defined as the class of simple (p_1, p_2)-Levinson conform matrices. This class incorporates, for instance, semiseparable, band, companion, arrowhead and many other matrices. For this class, we will derive a solver of complexity $O(p_1 p_2 n)$. The system solver is written inductively, and uses in every step k, the solution of a so-called kth order Yule-Walker-like equation. The algorithm obtained first has complexity $O(p_1 p_2 n^2)$. Based, however on the specific structure of the simple (p_1, p_2)-Levinson conform matrices, we will be able to further reduce the complexity of the presented method, and get an order $O(p_1 p_2 n)$ algorithm.

Different examples of matrices are given for this algorithm. Examples are presented for: general dense matrices, upper triangular matrices, higher order generator semiseparable matrices, quasiseparable matrices, Givens-vector representable semiseparable matrices, band matrices, companion matrices, confederate matrices, arrowhead matrices, fellow matrices and many more.

Finally, the relation between this method and an upper triangular factorization of the original matrix is given and also details concerning possible look ahead methods are presented.

Key words. Levinson, Yule-Walker, look-ahead, system solving, Levinson conform matrices

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