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# EFFICIENT DEFLATION METHODS APPLIED TO 3-D BUBBLY FLOW PROBLEMS*

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**Abstract.** For various applications, it is well-known that deflated ICCG is an efficient method to solve linear systems with an invertible coefficient matrix. Tang and Vuik [J. Comput. Appl. Math., 206 (2007), pp. 603–614] proposed two equivalent variants of this deflated method, which can also solve linear systems with singular coefficient matrices that arise from the discretization of the Poisson equation with Neumann boundary conditions and discontinuous coefficients. In this paper, we also consider the original variant of DICCG in Vuik, Segal, and Meijerink [J. Comput. Phys., 152 (1999), pp. 385–403], that already proved its efficiency for invertible coefficient matrices. This variant appears to be theoretically equivalent to the first two variants, so that they all have the same convergence properties. Moreover, we show that the associated coarse linear systems within these variants can be solved both directly and iteratively. In applications with large grid sizes, the method with the iterative coarse solver can be substantially more efficient than the one with the standard direct coarse solver.

Additionally, the results for stationary numerical experiments of Tang and Vuik [J. Comput. Appl. Math., 206 (2007), pp. 603–614] have only been given in terms of number of iterations. After discussing some implementation issues, we show in this paper that deflated ICCG is considerably faster than ICCG in the most test cases, by taking the computational time into account as well. Other 3-D time-dependent numerical experiments with falling droplets in air and rising air bubbles in water are performed, in order to show that deflated ICCG is also more efficient than ICCG in these cases, considering both the number of iterations and computational time.

**Key words.** deflation, conjugate gradient method, preconditioning, Poisson equation, symmetric positive semi-definite matrices, bubbly flow problems, inner-outer iterations

**AMS subject classifications.** 65F10, 65F50, 65N22

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