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34 Factorization of the hypergeometric-type difference equation on the uniform lattice. *R. Álvarez-Nodarse, N. M. Atakishiyev, and R. S. Costas-Santos*.

Abstract.
We discuss factorization of the hypergeometric-type difference equations on the uniform lattices and show how one can construct a dynamical algebra, which corresponds to each of these equations. Some examples are exhibited, in particular, we show that several models of discrete harmonic oscillators, previously considered in a number of publications, can be treated in a unified form.

Key Words.
discrete polynomials, factorization method, discrete oscillators

AMS(MOS) Subject Classifications.
33C45, 33C90, 39A13

51 Recent results concerning dynamic equations on time scales. *Lynn Erbe and Allan Peterson*.

Abstract.
We discuss a number of recent results for second order linear and nonlinear dynamic equations on time scales.

Key Words.
measure chains, Riccati equation, oscillation, nonoscillation, time scales

AMS(MOS) Subject Classifications.
34B10, 39A10

71 Langenhop's inequality and applications for dynamic equations. *B. Kaymakçalan and A. Zafer*.

Abstract.
A Langenhop-type inequality is given for dynamic equations on time scales. This result is further employed to obtain lower bounds for solutions of certain dynamic equations. As an application, usage of the derived Langenhop's inequality in determining the oscillatory behavior of a damped second order delay dynamic equation is illustrated. The results obtained are important in the qualitative sense.

Key Words.
Langenhop inequality, time scale, lower bounds, oscillation

AMS(MOS) Subject Classifications.
26D20, 34A40, 34K11

78 On difference schemes for quasilinear evolution problems. *Rita Meyer-Spasche*.

Abstract.
We review several methods leading to variable-coefficient schemes and/or to exact difference schemes for ordinary differential equations (error elimination; functional fitting; Principle of Coherence). Necessary and suffient conditions are given for t-independence of fitted RK coefficients. Conditions for τ-independence are investigated, τ the time-step. The theory is illustrated by examples. In particular, examples are given for non-uniqueness of exact schemes and for efficient difference schemes

based on exact schemes and well suited for highly oscillatory ordinary differential systems or for parabolic equations with blow-up solutions.

Key Words.
difference schemes, time stepping, nonstandard schemes, exact schemes, exponential fitting, functional fitting, Runge-Kutta, collocation methods, review

AMS(MOS) Subject Classifications.
65L05, 65M06, 65P99

Abstract.
For any sequence $\{a_k\}$ satisfying $0 < a_1 \leq a_2 \leq \ldots$ and $|a_k - k| \leq \text{Const}$ we find the Stieltjes representation of the function

$$z \mapsto \frac{\log P(z)}{z \operatorname{Log} z},$$

where P denotes the canonical product of genus 1 having $\{-a_k\}$ as its zero set.
We also find conditions on the zeros (e.g. $a_k \in [k, k+1]$ for $k \geq 1$) in order that the function

$$z \mapsto \frac{-\log P(z) + z \log P(1)}{z \operatorname{Log} z}$$

be a Pick function. We find the corresponding representation in terms of a positive density on the negative axis. We thereby generalize earlier results about the Γ-function. We also show that another related function is a Pick function.

Key Words.
pick function, canonical product, integral representation

AMS(MOS) Subject Classifications.
30E20, 30D15, 30E15, 33B15

Abstract.
This article surveys the classical orthogonal polynomial systems of the Hahn class, which are solutions of second-order differential, difference or q-difference equations.
Orthogonal families satisfy three-term recurrence equations. Example applications of an algorithm to determine whether a three-term recurrence equation has solutions in the Hahn class—implemented in the computer algebra system *Maple*—are given. Modifications of these families, in particular associated orthogonal systems, satisfy fourth-order operator equations. A factorization of these equations leads to a solution basis.

Key Words.
orthogonal polynomials, Hahn class, differential equations, difference equations, q-difference equations, hypergeometric functions, factorization of operator polynomials, computer algebra, Maple

AMS(MOS) Subject Classifications.
33C45, 33C20, 33D45, 33D15, 39A70

124 Left-definite variations of the classical Fourier expansion theorem. *L. L. Littlejohn and A. Zettl*.

Abstract.
In a recent paper, Littlejohn and Wellman developed a general left-definite theory for arbitrary self-adjoint operators in a Hilbert space that are bounded below by a positive constant. We apply this theory and construct the sequences of left-definite Hilbert spaces $\{H_n\}_{n\in\mathbb{N}}$ and left-definite self-adjoint operators $\{A_n\}_{n\in\mathbb{N}}$ associated with the classical, regular self-adjoint boundary value problem consisting of the Fourier equation with periodic boundary conditions. As a particular consequence of our analysis, we obtain a Fourier expansion theorem in each left-definite space H_n as well as an explicit representation of the domain of $A^{n/2}$ for each positive integer n.

Key Words.
self-adjoint operator, Hilbert space, left-definite Hilbert space, left-definite operator, regular self-adjoint boundary value problem, Fourier series

AMS(MOS) Subject Classifications.
34B24, 33B10

140 An algebra of integral operators. *Sergei K. Suslov*.

Abstract.
We introduce an algebra of integral operators related to a model of the q-harmonic oscillator and investigate some of its properties.

Key Words.
integral operators, divided difference operators, the continuous q-Hermite polynomials, generating functions, Poisson kernel, bilinear generating functions, q-harmonic oscillators

AMS(MOS) Subject Classifications.
33D45, 42C10, 45E10

156 Periodic points of some algebraic maps. *Valery G. Romanovski*.

Abstract.
We study the local dynamics of maps $f(z) = -z - \sum_{n=1}^{\infty} \alpha_n z^{n+1}$, where $f(z)$ is an irreducible branch of the algebraic curve

$$z + w + \sum_{i+j=n} a_{ij} z^i w^j = 0.$$

We show that the center and cyclicity problems have simple solutions when n is odd. For the case of even n some partial results are obtained.

Key Words.
discrete dynamical systems, polynomial maps, periodic points

AMS(MOS) Subject Classifications.
37F10, 58F, 13P

163 The difference equation related to the problem of the hydrogen atom in a strong magnetic field. *Marko Robnik and Valery G. Romanovski*.

Abstract.
We study the Schrödinger equation for the hydrogen atom in an arbitrarily strong magnetic field in two dimensions, which is an integrable and separable system. The energy spectrum is very interesting as it has infinitely many accumulation points located at the values of the Landau energy levels of a free electron in the uniform magnetic field. In the polar coordinates the canonical (not kinetic!) angular momentum has a precise eigenvalue and we have the one dimensional radial Schrödinger equation which is an ordinary second order differential equation whose analytic exact solution is unknown. The problem is reduced to a linear three-term recurrence difference equation whose solution is unknown. We describe the qualitative properties of the energy spectrum and propose a semi-analytic method to numerically calculate the eigenenergies.

Key Words.
hydrogen atom, strong magnetic field

AMS(MOS) Subject Classifications.
81V45, 81Q10