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PICK FUNCTIONS RELATED TO ENTIRE FUNCTIONS HAVING NEGATIVE ZEROS*

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Abstract. For any sequence $\{a_k\}$ satisfying $0 < a_1 \leq a_2 \leq \ldots$ and $|a_k - k| \leq$ Const we find the Stieltjes representation of the function

$$z \mapsto \frac{\log P(z)}{z \operatorname{Log} z},$$

where P denotes the canonical product of genus 1 having $\{-a_k\}$ as its zero set.

We also find conditions on the zeros (e.g. $a_k \in [k, k+1]$ for $k \geq 1$) in order that the function

$$z \mapsto \frac{-\log P(z) + z \log P(1)}{z \operatorname{Log} z}$$

be a Pick function. We find the corresponding representation in terms of a positive density on the negative axis. We thereby generalize earlier results about the Γ-function. We also show that another related function is a Pick function.

Key words. pick function, canonical product, integral representation

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