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DERIVATION OF HIGH-ORDER SPECTRAL METHODS FOR TIME-DEPENDENT PDE USING MODIFIED MOMENTS*

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In memory of Gene Golub

Abstract. This paper presents a reformulation of Krylov Subspace Spectral (KSS) Methods, which build on Gene Golub's many contributions pertaining to moments and Gaussian quadrature, to produce high-order accurate approximate solutions to variable-coefficient time-dependent PDE. This reformulation serves two useful purposes. First, it more clearly illustrates the distinction between KSS methods and existing Krylov subspace methods for solving stiff systems of ODE arising from discretizions of PDE. KSS methods rely on perturbations of Krylov subspaces in the direction of the data, and therefore rely on directional derivatives of nodes and weights of Gaussian quadrature rules. Second, because these directional derivatives allow KSS methods to be described in terms of operator splittings, they facilitate stability analysis. It will be shown that under reasonable assumptions on the coefficients of the problem, certain KSS methods are unconditionally stable. This paper also discusses preconditioning similarity transformations that allow more general problems to benefit from this property.

Key words. spectral methods, Gaussian quadrature, variable-coefficient, Lanczos method, stability, heat equation, wave equation

AMS subject classifications. 65M12, 65M70, 65D32

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