Electronic Transactions on Numerical Analysis
Volume 3, 1995

Contents

Abstract. We give the solution of a discrete least squares approximation problem in terms of orthonormal polynomial vectors with respect to a discrete inner product. The degrees of the polynomial elements of these vectors can be different. An algorithm is constructed computing the coefficients of recurrence relations for the orthonormal polynomial vectors. In case the weight vectors are prescribed in points on the real axis or on the unit circle, variants of the original algorithm can be designed which are an order of magnitude more efficient. Although the recurrence relations require all previous vectors to compute the next orthonormal polynomial vector, in the real or the unit-circle case only a fixed number of previous vectors are required. As an application, we approximate a vector-valued function by a vector rational function in a linearized least squares sense.

Key words. orthonormal polynomial vectors, least squares approximation, vector rational approximation.

AMS(MOS) subject classification. 42C05, 30E10, 65D10, 41A28, 41A20.

Files. vol.3.1995/pp1-23.dir/pp1-23.ps.

Forward References.

Abstract. Different types of synchronous and asynchronous two-stage multisplitting algorithms for the solution of linear systems are analyzed. The different algorithms which appeared in the literature are reviewed, and new ones are presented. Convergence properties of these algorithms are studied when the matrix in question is either monotone or an H-matrix. Relaxed versions of these algorithms are also studied. Computational experiments on a shared memory multiprocessor vector computer are presented.

Key words. asynchronous methods, two-stage iterative methods, linear systems, multisplittings, parallel algorithms.

AMS(MOS) subject classifications. 65F10, 65F15.

Files. vol.3.1995/pp24-38.dir/pp24-38.ps.

Forward References. vol.4.1996/pp1-13.dir/pp1-13.html, vol.5.1997/pp48-61.dir/pp48-61.html, vol.12.2001/pp88-112.dir/pp88-112.html.

Abstract. This paper is concerned with the singular values and vectors of a product $M_m = A_1 A_2 \cdots A_m$ of matrices of order n. The chief difficulty with computing

them directly from M_m is that with increasing m the ratio of the small to the large singular values of M_m may fall below the rounding unit, so that the former are computed inaccurately. The solution proposed here is to compute recursively the factorization $M_m = QRP^{\mathrm{T}}$, where Q is orthogonal, R is a graded upper triangular, and P^{T} is a permutation.

Key words. QR decomposition, singular value decomposition, graded matrix, matrix product.

AMS(MOS) subject classification. 65F30.

Files. vol.3.1995/pp39-49.dir/pp39-49.ps.

Forward References. vol.5.1997/pp29-47.dir/pp29-47.html.

50 Adaptive k-Step iterative methods for nonsymmetric systems of linear equations. *Thomas A. Manteuffel, Gerhard Starke, and Richard S. Varga.*

Abstract. This study is concerned with k-step methods for the iterative solution of nonsymmetric systems of real linear equations. These are generalizations of the Chebyshev (2-step) iteration, with the potential for faster convergence in cases where the spectrum of the underlying coefficient matrix is not approximated well by an ellipse. We investigate the problem of optimizing the associated (asymptotic) convergence factor with respect to a finite number of points (e.g., eigenvalue estimates obtained from using the Arnoldi process). We formulate this minimization problem as an optimization problem with constraints and propose an algorithm to compute near-best k-step parameters. The computational advantages of the Chebyshev method, such as avoidance of inner products, the implementation as an adaptive method, and the simplicity of the overall scheme, carry over to the case $k > 2$.

Key words. k-step method, Chebyshev method, adaptive implementation, polynomial iteration.

AMS(MOS) subject classification. 65F10.

Files. vol.3.1995/pp50-65.dir/pp50-65.ps.

Forward References.

66 Minimal Gerschgorin sets for partitioned matrices II. The spectral conjecture. *Alan Krautstengl and Richard S. Varga.*

Abstract. In an earlier paper from 1970, entitled “Minimal Gerschgorin sets for partitioned matrices,” a Spectral Conjecture, related to norms and spectral radii of special partitioned matrices, was stated, this conjecture being at the heart of the sharpness of the boundaries of the associated minimal Gerschgorin sets under partitioning. In this paper, this Spectral Conjecture is affirmatively settled, and is applied to the sharpness of the minimal Gerschgorin set in the special case when the block-diagonal entries are null matrices. The paper following this article then makes use of the proof of the Spectral Conjecture to obtain the general sharpness of the boundaries of the associated minimal Gerschgorin sets for partitioned matrices.

Key words. minimal Gerschgorin sets, partitioned matrices, monotonicity.

AMS(MOS) subject classification. 15A18.

Files. vol.3.1995/pp66-82.dir/pp66-82.ps.

Forward References.

83 Minimal Gerschgorin sets for partitioned matrices III. Sharpness of boundaries and monotonicity as a function of the partition. *Richard S. Varga and Alan Krautstengl.*

Abstract. Making use, from the preceding paper, of the affirmative solution of the Spectral Conjecture, it is shown here that the general boundaries, of the minimal Gerschgorin sets for partitioned matrices, are sharp, and that monotonicity of these minimal Gerschgorin sets, as a function of the partitionings, is obtained. These results extend and sharpen an earlier paper from 1970 on this topic.

Key words. minimal Gerschgorin sets, partitioned matrices, monotonicity.

AMS(MOS) subject classification. 15A18.

Files. vol.3.1995/pp83-95.dir/pp83-95.ps.

Forward References.

96 Gaussian quadrature for matrix valued functions on the unit circle. *Ann Sinap.*

Abstract. The Gaussian quadrature formulas for matrix valued functions on the unit circle are described. It is shown how the eigenvalues and eigenvectors of a unitary lower block Hessenberg matrix can be used to compute an approximation of a given matrix integral on the unit circle. A parallel algorithm for this purpose has been implemented on a IBM SP1 and some examples are worked out.

Key words. orthogonal matrix polynomials, block Hessenberg matrices, quadrature, parallel algorithm.

AMS(MOS) subject classification. 42C05, 41A55, 47A56, 65D32, 65Y05.

Files. vol.3.1995/pp96-115.dir/pp96-115.ps.

Forward References.

116 On the correctness of some bisection-like parallel eigenvalue algorithms in floating point arithmetic. *James W. Demmel, Inderjit Dhillon, and Huan Ren.*

Abstract. Bisection is a parallelizable method for finding the eigenvalues of real symmetric tridiagonal matrices, or more generally symmetric acyclic matrices. Ideally, one would like an implementation that was simultaneously parallel, load balanced, devoid of communication, capable of running on networks of heterogenous workstations, and of course correct. But this is surprisingly difficult to achieve. The reason is that bisection requires a function Count(x) which counts the number of eigenvalues less than x. In exact arithmetic Count(x) is a monotonic increasing function of x, and the logic of the algorithm depends on this. However, monotonicity can fail, and incorrect eigenvalues may be computed, because of roundoff or as a result of using networks of heterogeneous parallel processors. We illustrate this problem, which even arises in some serial algorithms like the EISPACK routine `bisect`, and show several ways to fix it. One of these ways has been incorporated into the ScaLAPACK library.

Key words. symmetric eigenvalue problem, parallel algorithms, monotonicity, correctness, floating point.

AMS(MOS) subject classification. 65F15, 65Y05.

Files. vol.3.1995/pp116-149.dir/pp116-149.ps.

Forward References.

150 First-order system least squares for velocity-vorticity-pressure form of the Stokes equations. *Zhiqiang Cai, Thomas A. Manteuffel, and Stephen F. McCormick.*

Abstract. In this paper, we study the least-squares method for the generalized Stokes equations (including linear elasticity) based on the velocity-vorticity-pressure formulation in $d = 2$ or 3 dimensions. The least-squares functional is defined in terms of the sum of the L^2- and H^{-1}-norms of the residual equations, which is similar to that in [7], but weighted appropriately by the Reynolds number (Poisson ratio). Our approach for establishing ellipticity of the functional does not use ADN theory, but is founded more on basic principles. We also analyze the case where the H^{-1}-norm in the functional is replaced by a discrete functional to make the computation feasible. We show that the resulting algebraic equations can be preconditioned by well-known techniques uniformly well in the Reynolds number (Poisson ratio).
Key words. least squares, Stokes, elasticity.

AMS(MOS) subject classification. 65F10, 65F30.

Files. vol.3.1995/pp150-159.dir/pp150-159.ps.

Forward References. vol.6.1997/pp35-43.dir/pp35-43.html, vol.6.1997/pp44-62.dir/pp44-62.html.

160 A parallel GMRES version for general sparse matrices. *Jocelyne Erhel.*

Abstract. This paper describes the implementation of a parallel variant of GMRES on Paragon. This variant builds an orthonormal Krylov basis in two steps: it first computes a Newton basis then orthogonalises it. The first step requires matrix-vector products with a general sparse unsymmetric matrix and the second step is a QR factorisation of a rectangular matrix with few long vectors. The algorithm has been implemented for a distributed memory parallel computer. The distributed sparse matrix-vector product avoids global communications thanks to the initial setup of the communication pattern. The QR factorisation is distributed by using Givens rotations which require only local communications. Results on an Intel Paragon show the efficiency and the scalability of our algorithm.

Key words. GMRES, parallelism, sparse matrix, Newton basis.

AMS(MOS) subject classification. 65F10, 65F25, 65F50.

Files. vol.3.1995/pp160-176.dir/pp160-176.ps.

Forward References.