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degree n. To this end, we show that the corresponding linear mapping is represented by the eigenvector matrix of an explicitly known diagonal plus upper triangular semiseparable matrix. The method is based on a new efficient algorithm for computing the eigendecomposition of such a matrix. Using fast summation techniques, the eigenvectors of an $n \times n$ matrix can be computed explicitly with $\mathcal{O}\left(n^2\right)$ arithmetic operations and the eigenvector matrix can be applied to an arbitrary vector at cost $\mathcal{O}\left(n \log n\right)$. All algorithms are accurate up to a prefixed accuracy ε. We provide brief numerical results.

Key Words.
Gegenbauer polynomials, polynomial transforms, semiseparable matrices, eigendecomposition, spectral divide-and-conquer methods

AMS(MOS) Subject Classifications.
42C10, 42C20, 15A18, 15A23, 15A57, 65T50, 65Y20

Abstract.
In this paper, we investigate the regularization properties of Tikhonov regularization with a sparsity (or Besov) penalty for the inversion of nonlinear operator equations. We propose an a posteriori parameter choice rule that ensures convergence in the used norm as the data error goes to zero. We show that the method of surrogate functionals will at least reconstruct a critical point of the Tikhonov functional. Finally, we present some numerical results for a nonlinear Hammerstein equation.

Key Words.
inverse problems, sparsity

AMS(MOS) Subject Classifications.
65J15, 65J20, 65J22

Dedicated to Prof. K. Maleknejad on the occasion of his 62th birthday.

Abstract.
We present a method and experimental results for approximate solution of nonlinear Volterra-Fredholm integral equations by double exponential (DE) transformation based on the sinc collocation method. It is well known that by applying DE transformation the rate of convergence $O(\exp(-cN/\log N))$ is attained, where N is a parameter representing the number of terms of the sinc expansion. The purpose of this paper is to develop the work carried out in 2005 by Muhammad et al. [J. Comput. Appl. Math., 177 (2005), pp. 269–286], for the numerical solution of two dimensional nonlinear Volterra-Fredholm integral equations. We design a numerical scheme for these equations based on the sinc collocation method incorporated with the DE transformation. A new error estimation by truncation is also obtained which is shown to have an exponential order of convergence as in Muhammad et al. (op. cit.). Finally, the reliability and efficiency of the proposed scheme are demonstrated by some numerical experiments.

Key Words.
integral equation, sinc collocation method, double exponential transformation, Volterra-Fredholm integral equation, error estimation

AMS(MOS) Subject Classifications.
65D32, 45G10

88 Minimal degree rational unimodular interpolation on the unit circle. *Christer Glader*.

Abstract.
We consider an interpolation problem with n distinct nodes $z_1, \ldots, z_n$ and n interpolation values $w_1, \ldots, w_n$, all on the complex unit circle, and seek interpolants $b(z)$ of minimal degree in the class consisting of ratios of finite Blaschke products. The focus is on the so-called damaged cases where the interpolant of minimal degree is non-uniquely determined. This paper is a continuation of the work in Glader [Comput. Methods Funct. Theory, 6 (2006), pp. 481–492], which treated the uniquely solvable fragile and elastic cases.

Key Words.
rational interpolation, Blaschke product, Nevanlinna parametrization

AMS(MOS) Subject Classifications.
30D50, 35E05

107 A weakly over-penalized symmetric interior penalty method. *Susanne C. Brenner, Luke Owens, and Li-Yeng Sung*.

Abstract.
We introduce a new symmetric interior penalty method for symmetric positive definite second order elliptic boundary value problems, where the jumps across element boundaries are weakly over-penalized. Error estimates are derived in the energy norm and the L_2 norm for both conforming and nonconforming meshes. Numerical results illustrating the performance of the method are also presented.

Key Words.
symmetric interior penalty method, weak over-penalization

AMS(MOS) Subject Classifications.
65N30

128 The dynamical motion of the zeros of the partial sums of e^z, and its relationship to discrepancy theory. *Richard S. Varga, Amos J. Carpenter, and Bryan W. Lewis*.

Dedicated to Edward B. Saff on his 64th birthday, January 2, 2008.

Abstract.
With $s_n(z) := \sum_{k=0}^{n} z^k/k!$ denoting the n-th partial sum of e^z, let its zeros be denoted by $\{z_{k,n}\}_{k=1}^{n}$ for any positive integer n. If θ_1 and θ_2 are any angles with $0 < \theta_1 < \theta_2 < 2\pi$, let Z_{θ_1,θ_2} be the associated sector, in the z-plane, defined by

$$Z_{\theta_1,\theta_2} := \{z \in \mathbb{C} : \theta_1 \leq \arg z \leq \theta_2\}.$$

If $\#\left(\{z_{k,n}\}_{k=1}^{n} \bigcap Z_{\theta_1,\theta_2}\right)$ represents the number of zeros of $s_n(z)$ in the sector Z_{θ_1,θ_2}, then Szegő showed in 1924 that

$$\lim_{n\to\infty} \frac{\#\left(\{z_{k,n}\}_{k=1}^{n} \bigcap Z_{\theta_1,\theta_2}\right)}{n} = \frac{\phi_2 - \phi_1}{2\pi},$$

where ϕ_1 and ϕ_2 are defined in the text. The associated *discrepancy function* is defined by

$$\mathrm{disc}_n(\theta_1, \theta_2) := \#\left(\{z_{k,n}\}_{k=1}^{n} \bigcap Z_{\theta_1,\theta_2}\right) - n\left(\frac{\phi_2 - \phi_1}{2\pi}\right).$$

One of our new results shows, for any θ_1 with $0 < \theta_1 < \pi$, that

$$\mathrm{disc}_n(\theta_1, 2\pi - \theta_1) \sim K \log n, \quad \text{as} \quad n \to \infty,$$

where K is a positive constant, depending only on θ_1. Also new in this paper is a study of the *cyclical nature* of $\mathrm{disc}_n(\theta_1, \theta_2)$, as a function of n, when $0 < \theta_1 < \pi$ and $\theta_2 = 2\pi - \theta_1$. An upper bound for the approximate cycle length, in this case, is determined in terms of ϕ_1. All this can be viewed in our *Interactive Supplement*, which shows the dynamical motion of the (normalized) zeros of the partial sums of e^z and their associated discrepancies.

Key Words.
partial sums of e^z, Szegő curve, discrepancy function

AMS(MOS) Subject Classifications.
30C15, 30E15

144 A parallel QR-factorization/solver of quasiseparable matrices. *Raf Vandebril, Marc Van Barel, and Nicola Mastronardi.*

Abstract.
This manuscript focuses on the development of a parallel QR-factorization of structured rank matrices, which can then be used for solving systems of equations. First, we will prove the existence of two types of Givens transformations, named rank decreasing and rank expanding Givens transformations. Combining these two types of Givens transformations leads to different patterns for annihilating the lower triangular part of structured rank matrices. How to obtain different annihilation patterns, for computing the upper triangular factor R, such as the $\vee$ and $\wedge$ pattern will be investigated. Another pattern, namely the X-pattern, will be used for computing the QR-factorization in a parallel way.
As an example of such a parallel QR-factorization, we will implement it for a quasiseparable matrix. This factorization can be run on 2 processors, with one step of intermediate communication in which one row needs to be sent from one processor to the other and back. Another example, showing how to deduce a parallel QR-factorization for a more general rank structure will also be discussed.
Numerical experiments are included for demonstrating the accuracy and speed of this parallel algorithm w.r.t. the existing factorization of quasiseparable matrices. Also some numerical experiments on solving systems of equations using this approach will be given.

Key Words.
parallel QR-factorization, structured rank matrices, quasiseparable matrix

AMS(MOS) Subject Classifications.
65F05

168 Calculation of minimum critical Reynolds number for laminar-turbulent transition in pipe flows. *Hidesada Kanda*.

Abstract.
This article describes the calculation of the minimum critical Reynolds number for laminar-turbulent transition in pipe flows. From the conclusions of our previous experimental study, it is clear that a transition occurs near the pipe inlet and the critical Reynolds number R_c takes the minimum value of about 2000 in the case of a straight pipe. Moreover, in our previous calculations of laminar entrance pipe flow, it was found that near the pipe inlet a large pressure gradient in the radial direction exists, which decreases as the Reynolds number Re increases. Thus, we have built a new transition macromodel to determine R_c using the effect of the radial pressure gradient. The calculated results were $R_c(min)$ = 3750 when the number of radial grid points $J0$ = 51 and 2200 when $J0$ = 101.

Key Words.
hydrodynamic stability, grid refinement, thermodynamics

AMS(MOS) Subject Classifications.
76E05, 65M50, 80A05

187 Low-rank iterative methods for projected generalized Lyapunov equations. *Tatjana Stykel*.

Abstract.
We generalize an alternating direction implicit method and the Smith method for large-scale projected generalized Lyapunov equations. Such equations arise in model reduction of descriptor systems. Low-rank versions of these methods are also presented, which can be used to compute low-rank approximations to the solution of projected generalized Lyapunov equations with low-rank symmetric, positive semidefinite right-hand side. Numerical examples are presented.

Key Words.
projected generalized Lyapunov equations, alternating direction implicit method, Smith method, low-rank approximation

AMS(MOS) Subject Classifications.
65F10, 65F30, 15A22, 15A24,

203 The automatic computation of second-order slope tuples for some nonsmooth functions. *Marco Schnurr*.

Abstract.
In this paper, we show how the automatic computation of second-order slope tuples can be performed. The algorithm allows for nonsmooth functions, such as $\varphi(x) = |u(x)|$ and $\varphi(x) = \max\{u(x), v(x)\}$, to occur in the function expression of the underlying function. Furthermore, we allow the function expression to contain functions given by two or more branches. By using interval arithmetic, second-order slope tuples provide verified enclosures of the range of the underlying function. We give some examples comparing range enclosures given by a second-order slope tuple with enclosures from previous papers.

Key Words.
slope tuple, interval analysis, automatic slope computation, range enclosure

AMS(MOS) Subject Classifications.
65G20, 65G99

224 A simplification of the Laplace method for double integrals. Application to the second Appell function. *José L. López and Pedro J. Pagola*.

Abstract.
The main difficulties in the Laplace method of asymptotic expansions of double integrals result from a change of variables. Generalizing previous work for simple integrals, we propose a variant of the method for double integrals, which avoids this change of variables and simplifies the computations. The calculation of the coefficients of the asymptotic expansion is remarkably simple. Moreover, the asymptotic sequence is as simple as in the standard Laplace's method: inverse powers of the asymptotic variable. A new asymptotic expansion of the second Appell's function $F_2(a, b, b', c, c'; x, y)$ for large b, b', c and c' is given as an illustration.

Key Words.
asymptotic expansions of integrals, Laplace method for double integrals, second Appell function

AMS(MOS) Subject Classifications.
41A60, 41A58, 33C65

237 Asymptotic behavior for numerical solutions of a semilinear parabolic equation with a nonlinear boundary condition. *Nabongo Diabate and Théodore K. Boni*.

Abstract.
This paper concerns the study of the numerical approximation for the following initial-boundary value problem,

$$
\begin{aligned}
&u_t = u_{xx} - au^p, \quad 0 < x < 1, \quad t > 0,\\
&u_x(0,t) = 0, \quad u_x(1,t) + bu^q(1,t) = 0, \quad t > 0,\\
&u(x,0) = u_0(x) \geq 0, \quad 0 \leq x \leq 1,
\end{aligned}
$$

where $a > 0$, $b > 0$ and $p > q > 1$. We show that the solution of a semidiscrete form of the initial value problem above goes to zero as t approaches infinity and give its asymptotic behavior. We provide some numerical experiments that illustrate our analysis.

Key Words.
semidiscretizations, semilinear parabolic equation, asymptotic behavior, convergence

AMS(MOS) Subject Classifications.
35B40, 35B50, 35K60, 65M06

247 Numerical blow-up solutions for some semilinear heat equations. *Firmin K. N'Gohisse and Théodore K. Boni*.

Abstract.
This paper concerns the study of the numerical approximation for the following

initial-boundary value problem,

$$u_t = u_{xx} + \frac{b}{x}u_x + u^p, \quad x \in (0,1), \quad t \in (0,T),$$
$$u_x(0,t) = 0, \quad u(1,t) = 0, \quad t \in (0,T),$$
$$u(x,0) = u_0(x), \quad x \in [0,1],$$

where $b > 0$ and $p > 1$. We give some conditions under which the solution of a semidiscrete form of the above problem blows up in a finite time and estimate its semidiscrete blow-up time. Under some assumptions, we also show that the semidiscrete blow-up time converges to the continuous blow-up time when the mesh size goes to zero. Finally, we give some numerical results to illustrate our analysis.

Key Words.
semidiscretizations, discretizations, semilinear heat equations, semidiscrete blow-up time

AMS(MOS) Subject Classifications.
35B40, 35K65, 65M06

Abstract.
This paper presents a generalization of the optimal finite-difference perfectly matched layer (PML) approach to isotropic elasticity. It allows the use of methods of rational approximation theory for a clever choice of discretization parameters in order to essentially reduce reflection coefficients for a wide range of incident angles while using a small number of grid points.

Key Words.
artificial boundary conditions, optimal grids, perfectly matched layers, finite-difference schemes, rational approximation

AMS(MOS) Subject Classifications.
65N06, 74C02

Abstract.
In the present paper, we obtain quadrature rules for Bernstein measures on $[-1,1]$, having a fixed number of nodes and weights such that they exactly integrate functions in the linear space of polynomials with real coefficients.

Key Words.
quadrature rules, orthogonal polynomials, measures on the real line, Bernstein measures, Chebyshev polynomials

AMS(MOS) Subject Classifications.
33C47, 42C05

291 A convergent adaptive finite element method with optimal complexity. *Roland Becker, Shipeng Mao, and Zhong-Ci Shi*.

Abstract.
In this paper, we introduce and analyze a simple adaptive finite element method for second order elliptic partial differential equations. The marking strategy depends on whether the data oscillation is sufficiently small compared to the error estimator in the current mesh. If the oscillation is small compared to the error estimator, we mark as many edges such that their contributions to the local estimator are at least a fixed proportion of the global error estimator (bulk criterion for the estimator). Otherwise, we reduce the oscillation by marking sufficiently many elements, such that the oscillations of the marked cells are at least a fixed proportion of the global oscillation (bulk criterion for the oscillation). This marking strategy guarantees a strict reduction of the error augmented by the oscillation term. Both convergence rates and optimal complexity of the adaptive finite element method are established, with an explicit expression of the constants in the estimates.

Key Words.
adaptive finite element method, a posteriori error estimator, convergence rate, optimal computational complexity

AMS(MOS) Subject Classifications.
65N12, 65N15, 65N30, 65N50

305 Stability analysis of fast numerical methods for Volterra integral equations. *G. Capobianco, D. Conte, I. Del Prete, and E. Russo*.

Abstract.
In this paper the stability properties of fast numerical methods for Volterra integral equations of Hammerstein type with respect to significant test equations are investigated.

Key Words.
fast numerical methods, Volterra Runge-Kutta methods, collocation methods, numerical stability

AMS(MOS) Subject Classifications.
65R20, 45D05, 44A35, 44A10

323 On algebraic multilevel methods for non-symmetric systems — convergence results. *Christian Mense and Reinhard Nabben*.

Abstract.
We analyze algebraic multilevel methods applied to non-symmetric M-matrices. Two types of multilevel approximate block factorizations are considered. The first one is related to the AMLI method. The second method is the multiplicative counterpart of the AMLI approach which we call the multiplicative algebraic multilevel (MAMLI) method. The MAMLI method is closely related to certain geometric and algebraic multigrid methods, such as the AMGr method. Although these multilevel methods work very well in practice for many problems, not much is known about theoretical convergence properties for non-symmetric problems. Here, we establish convergence results and comparison results between AMLI and MAMLI multilevel methods applied to non-symmetric M-matrices.

Key Words.
algebraic multilevel methods, multilevel approximate block factorizations, algebraic multigrid methods, AMLI method

AMS(MOS) Subject Classifications.
65F10, 65F50, 65N22

346 Parameter-uniform fitted operator B-spline collocation method for self-adjoint singularly perturbed two-point boundary value problems. *Mohan K. Kadalbajoo and Devendra Kumar*.

Abstract.
In this paper, we develop a B-spline collocation method for the numerical solution of a self-adjoint singularly perturbed boundary value problem of the form

$$-\epsilon(a(x)y')' + b(x)y(x) = f(x), \quad a(x) \geq a^* > 0,\ b(x) \geq b^* > 0,\ a'(x) \geq 0,$$
$$y(0) = \alpha, \quad y(1) = \beta.$$

We construct a fitting factor and use the B-spline collocation method, which leads to a tridiagonal linear system. The method is analyzed for parameter-uniform convergence. Several numerical examples are reported which demonstrate the efficiency of the proposed method.

Key Words.
B-spline collocation method, self-adjoint singularly perturbed boundary value problem, parameter-uniform convergence, boundary layer, fitted operator method

AMS(MOS) Subject Classifications.
34D15, 30E25, 20B40

359 An overlapping additive Schwarz-Richardson method for monotone nonlinear parabolic problems. *M. Munteanu and L. F. Pavarino*.

Abstract.
We construct and study a scalable overlapping Additive Schwarz-Richardson (ASR) algorithm for monotone nonlinear parabolic problems discretized implicitly in time. At each time step, the Additive Schwarz preconditioner is built using the linear part of the nonlinear operator, partitioning the domain of the problem into overlapping subdomains, solving local problems on these subdomains and solving an additional coarse problem associated with the subdomain mesh. This preconditioner is then applied to the nonlinear operator using a Richardson iteration. We prove first an abstract convergence result and then convergence rate estimates showing the scalability of the ASR algorithm. The results of numerical experiments in the plane confirm the theoretical estimates and illustrate the performance of the one and two-level ASR algorithm and in the presence of discontinuous coefficients in the parabolic operator.

Key Words.
monotone nonlinear parabolic problems, domain decomposition preconditioners, overlapping additive Schwarz, finite elements, implicit time discretizations

AMS(MOS) Subject Classifications.
65M55, 65H05

377 On the calculation of approximate Fekete points: the univariate case. *L. P. Bos and N. Levenberg*.

Abstract.

We discuss some theoretical aspects of the univariate case of the method recently introduced by Sommariva and Vianello [Comput. Math. Appl., to appear] for the calculation of approximate Fekete points for polynomial interpolation.

Key Words.
polynomial interpolation, Fekete points

AMS(MOS) Subject Classifications.
41A05, 42A15, 65D05

398 *Richard S. Varga, Ljiljana Cvetković, and Vladimir Kostić*. Approximation of the minimal Geršgorin set of a square complex matrix.

Abstract.

In this paper, we address the problem of finding a numerical approximation to the minimal Geršgorin set, $\Gamma^{\mathcal{R}}(A)$, of an irreducible matrix A in $\mathbb{C}^{n,n}$. In particular, boundary points of $\Gamma^{\mathcal{R}}(A)$ are related to a well-known result of Olga Taussky.

Key Words.
eigenvalue localization, Geršgorin theorem, minimal Geršgorin set

AMS(MOS) Subject Classifications.
15A18, 65F15

406 Fast wave propagation by model order reduction. *V. Pereyra and B. Kaelin*.

Abstract.

Large scale wave propagation simulation is currently achievable in reasonable turnaround times by using distributed computing in multiple cpu clusters. However, if one needs to perform many such simulations, as is the case in optimization, tomography, or seismic imaging, then the resources required are still prohibitive. Model order reduction of large dynamical systems has been successfully used in several application domains to paliate that problem and in this paper we explore one of its manifestations, Proper Orthogonal Decomposition, for wave propagation. We describe the method and show how it can be easily interfaced with two different high fidelity simulators. We exemplify its use on several problems of increasing complexity and size.

Key Words.
wave propagation, model order reduction, proper orthogonal decomposition

AMS(MOS) Subject Classifications.
65M99