

GEGENBAUER POLYNOMIALS AND SEMISEPARABLE MATRICES*

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Abstract. In this paper, we develop a new $\mathcal{O}(n \log n)$ algorithm for converting coefficients between expansions in different families of Gegenbauer polynomials up to a finite degree n . To this end, we show that the corresponding linear mapping is represented by the eigenvector matrix of an explicitly known diagonal plus upper triangular semiseparable matrix. The method is based on a new efficient algorithm for computing the eigendecomposition of such a matrix. Using fast summation techniques, the eigenvectors of an $n \times n$ matrix can be computed explicitly with $\mathcal{O}(n^2)$ arithmetic operations and the eigenvector matrix can be applied to an arbitrary vector at cost $\mathcal{O}(n \log n)$. All algorithms are accurate up to a prefixed accuracy ε . We provide brief numerical results.

Key words. Gegenbauer polynomials, polynomial transforms, semiseparable matrices, eigendecomposition, spectral divide-and-conquer methods

AMS subject classifications. 42C10, 42C20, 15A18, 15A23, 15A57, 65T50, 65Y20

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