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## STRUCTURED LOW RANK APPROXIMATIONS OF THE SYLVESTER RESULTANT MATRIX FOR APPROXIMATE GCDS OF BERNSTEIN BASIS POLYNOMIALS\*

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Abstract. A structured low rank approximation of the Sylvester resultant matrix S(f, g) of the Bernstein basis polynomials f = f(y) and g = g(y), for the determination of their approximate greatest common divisors (GCDs), is computed using the method of structured total least norm. Since the GCD of f(y) and g(y) is equal to the GCD of f(y) and  $\alpha g(y)$ , where  $\alpha$  is an arbitrary non-zero constant, it is more appropriate to consider a structured low rank approximation  $S(\tilde{f}, \tilde{g})$  of  $S(f, \alpha g)$ , where the polynomials  $\tilde{f} = \tilde{f}(y)$  and  $\tilde{g} = \tilde{g}(y)$  approximate the polynomials f(y) and  $\alpha g(y)$ , respectively. Different values of  $\alpha$  yield different structured low rank approximations  $S(f, \tilde{g})$ , and therefore different approximate GCDs. It is shown that the inclusion of  $\alpha$  allows to obtain considerably improved approximations, as measured by the decrease of the singular values  $\sigma_i$  of  $S(\tilde{f}, \tilde{g})$ , with respect to the approximation obtained when the default value  $\alpha = 1$  is used. An example that illustrates the theory is presented and future work is discussed.

Key words. Bernstein polynomials, structured low rank approximation, Sylvester resultant matrix.

AMS subject classifications. 15A12, 65F35.

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