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ENHANCEMENT OF KRYLOV SUBSPACE SPECTRAL METHODS BY BLOCK LANCZOS ITERATION*

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Dedicated to the memory of Gene H. Golub, 1932-2007

Abstract. This paper presents a modification of Krylov subspace spectral (KSS) methods, which build on the work of Golub, Meurant and others, pertaining to moments and Gaussian quadrature to produce high-order accurate approximate solutions to variable-coefficient time-dependent PDEs. Whereas KSS methods currently use Lanczos iteration to compute the needed quadrature rules, our modification uses block Lanczos iteration in order to avoid the need to compute two quadrature rules for each component of the solution, or use perturbations of quadrature rules. It will be shown that, under reasonable assumptions on the coefficients of the problem, a 1-node KSS method is unconditionally stable, and methods with more than one node are shown to possess favorable stability properties as well. Numerical results suggest that block KSS methods are significantly more accurate than their non-block counterparts.

Key words. Spectral methods, Gaussian quadrature, variable-coefficient, block Lanczos method, stability, heat equation.

AMS subject classifications. 65M12, 65M70, 65D32, 65F25.

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