Electronic Transactions on Numerical Analysis
Volume 33, 2008–2009

Contents

Abstract.
Some integrated models for ranking scientific publications together with authors and journals are presented and analyzed. The models rely on certain adjacency matrices obtained from the relationships between citations, authors and publications, which together give a suitable irreducible stochastic matrix whose Perron vector provides the ranking. Some perturbation theorems concerning the Perron vectors of nonnegative irreducible matrices are proved. These theoretical results provide a validation of the consistency and effectiveness of our models. Several examples are reported together with some results obtained on a real set of data.

Key Words.
page rank, Perron vector, perturbation results, impact factor

AMS Subject Classifications.
65F15

Abstract.
Krylov subspace methods are strongly related to polynomial spaces and their convergence analysis can often be naturally derived from approximation theory. Analyses of this type lead to discrete min-max approximation problems over the spectrum of the matrix, from which upper bounds on the relative Euclidean residual norm are derived. A second approach to analyzing the convergence rate of the GMRES method or the Arnoldi iteration, uses as a primary indicator the (1,1) entry of the inverse of $K_m^H K_m$ where K_m is the Krylov matrix, i.e., the matrix whose column vectors are the first m vectors of the Krylov sequence. This viewpoint allows us to provide, among other things, a convergence analysis for normal matrices using constrained convex optimization. The goal of this paper is to explore the relationships between these two approaches. Specifically, we show that for normal matrices, the Karush-Kuhn-Tucker (KKT) optimality conditions derived from the convex maximization problem are identical to the properties that characterize the polynomial of best approximation on a finite set of points. Therefore, these two approaches are

mathematically equivalent. In developing tools to prove our main result, we will give an improved upper bound on the distances of a given eigenvector from Krylov spaces.

Key Words.
Krylov subspaces, polynomials of best approximation, min-max problem, interpolation, convex optimization, KKT optimality conditions

AMS Subject Classifications.
65F10, 65F15

31 Using FGMRES to obtain backward stability in mixed precision. *M. Arioli and I. S. Duff*.

Abstract.
We consider the triangular factorization of matrices in single-precision arithmetic and show how these factors can be used to obtain a backward stable solution. Our aim is to obtain double-precision accuracy even when the system is ill-conditioned. We examine the use of iterative refinement and show by example that it may not converge. We then show both theoretically and practically that the use of FGMRES will give us the result that we desire with fairly mild conditions on the matrix and the direct factorization. We perform extensive experiments on dense matrices using MATLAB and indicate how our work extends to sparse matrix factorization and solution.

Key Words.
FGMRES, mixed precision arithmetic, hybrid method, direct factorization, iterative methods, large sparse systems, error analysis

AMS Subject Classifications.
65F05, 65F10, 65F50, 65G20, 65G50

45 A note on symplectic block reflectors. *Miloud Sadkane and Ahmed Salam*.

Abstract.
A symplectic block reflector is introduced. The parallel with the Euclidean block reflector is studied. Some important features of symplectic block reflectors are given. Algorithms to compute a symplectic block reflector that introduces a desired block of zeros into a matrix are developed.

Key Words.
skew-symmetric scalar product, symplectic Householder transformation, block algorithm, symplectic QR-factorization

AMS Subject Classifications.
65F30, 65F20, 15A57, 15A23

53 An extended block Arnoldi algorithm for large-scale solutions of the continuous-time algebraic Riccati equation. *M. Heyouni and K. Jbilou*.

Abstract.
We present a new iterative method for the computation of approximate solutions to large-scale continuous-time algebraic Riccati equations. The proposed method is a projection method onto an extended block Krylov subspace, which can be seen as a

sum of two block Krylov subspaces in A and A^{-1}. We give some theoretical results and present numerical experiments for large and sparse problems. These numerical tests show the efficiency of the proposed scheme as compared to the block Arnoldi and Newton-ADI methods.

Key Words.
block Arnoldi, extended block Krylov, low rank, Riccati equations

AMS Subject Classifications.
65F10, 65F30

63 Simple square smoothing regularization operators. *Lothar Reichel and Qiang Ye*.

Abstract.
Tikhonov regularization of linear discrete ill-posed problems often is applied with a finite difference regularization operator that approximates a low-order derivative. These operators generally are represented by a banded rectangular matrix with fewer rows than columns. They therefore cannot be applied in iterative methods that are based on the Arnoldi process, which requires the regularization operator to be represented by a square matrix. This paper discusses two approaches to circumvent this difficulty: zero-padding the rectangular matrices to make them square and extending the rectangular matrix to a square circulant. We also describe how to combine these operators by weighted averaging and with orthogonal projection. Applications to Arnoldi and Lanczos bidiagonalization-based Tikhonov regularization, as well as to truncated iteration with a range-restricted minimal residual method, are presented.

Key Words.
ill-posed problem, regularization operator, Tikhonov regularization, truncated iteration

AMS Subject Classifications.
65F10, 65F22, 65R32

84 Fast solution of a certain Riccati equation through Cauchy-like matrices. *Dario A. Bini, Beatrice Meini, and Federico Poloni*.

Abstract.
We consider a special instance of the algebraic Riccati equation $XCX - XE - AX + B = 0$ encountered in transport theory, where the $n \times n$ matrix coefficients A, B, C, E are rank structured matrices. The equation is reduced to unilateral form $A_1X^2+A_0X+A_{-1} = 0$ and solved by means of Cyclic Reduction (CR). It is shown that the matrices generated by CR are Cauchy-like with respect to a suitable singular operator and their displacement structure is explicitly determined. The application of the GKO algorithm provides a method for solving this Riccati equation in $O(n^2)$ arithmetic operations (ops) with quadratic convergence. The structured doubling algorithm is analyzed in the same framework and accelerated to $O(n^2)$ ops as well. In critical cases where convergence turns to linear, we present an adaptation of the

shift technique which allows us to get rid of the singularity. Numerical experiments and comparisons which confirm the effectiveness of the new approach are reported.

Key Words.
nonsymmetric algebraic Riccati equation, cyclic reduction, Cauchy matrix, matrix equation, fast algorithm, M-matrix

AMS Subject Classifications.
15A24, 65F05, 65H10

Abstract.
A nonlinear inverse scattering problem arising in microwave imaging is analyzed and numerically solved. In particular, the dielectric properties of an inhomogeneous object (i.e., the image to restore) are retrieved by means of its scattered microwave electromagnetic field (i.e., the input data) in a tomographic arrangement. From a theoretical point of view, the model gives rise to a nonlinear integral equation, which is solved by a deterministic and regularizing inexact Gauss-Newton method. At each step of the method, matrix strategies of numerical linear algebra are considered in order to reduce the computational (time and memory) load for solving the obtained large and structured linear systems. These strategies involve block decompositions, splitting and regularization, and super-resolution techniques. Some numerical results are given where the proposed algorithm is applied to recover high resolution images of the scatterers.

Key Words.
inverse scattering, microwave imaging, inexact-Newton methods, block decomposition, regularization

AMS Subject Classifications.
65F22, 65R32, 45Q05, 78A46

Abstract.
This paper proposes a new type of iteration for computing eigenvalues of semiseparable (plus diagonal) matrices based on a structured-rank factorization. Remarks on higher order semiseparability ranks are also made. More precisely, instead of the traditional QR iteration, a QH iteration is used. The QH factorization is characterized by a unitary matrix Q and a Hessenberg-like matrix H in which the lower triangular part is semiseparable (often called a lower semiseparable matrix). The Q factor of this factorization determines the similarity transformation of the QH method.
It is shown that this iteration is extremely useful for computing the eigenvalues of structured-rank matrices. Whereas the traditional QR method applied to semiseparable (plus diagonal) and Hessenberg-like matrices uses similarity transformations involving $2p(n-1)$ Givens transformations (where p denotes the semiseparability rank), the QH iteration only needs $p(n-1)$ Givens transformations, which is comparable to the generalized Hessenberg (symmetric band) situation having p subdiagonals. It is also shown that this method can in some sense be interpreted as an extension of the traditional QR method for Hessenberg matrices, i.e., the traditional

case also fits into this framework. It is also shown that this iteration exhibits an extra type of convergence behavior compared to the traditional QR method.
The algorithm is implemented in an implicit way, based on the Givens-weight representation of the structured rank matrices. Numerical experiments show the viability of this approach. The new approach yields better complexity and more accurate results than the traditional QR method.

Key Words.
QH algorithm, structured rank matrices, implicit computations, eigenvalue, QR algorithm, rational QR iteration

AMS Subject Classifications.
65F05

Abstract.
For many applications arising in system theory, it is important to know the structure and the dimension of the null spaces of certain structured matrices, such as Hankel and Toeplitz matrices. In this paper, we describe an algorithm based on the generalized Schur algorithm that computes the kernel of Toeplitz and Hankel matrices.

Key Words.
null space, Toeplitz matrix, Hankel matrix, generalized Schur algorithm

AMS Subject Classifications.
15A15, 15A09, 15A23

Abstract.
In this paper, we consider a class of hierarchically rank structured matrices that includes some of the hierarchical matrices occurring in the literature, such as hierarchically semiseparable (HSS) and certain $\mathcal{H}^2$-matrices. We describe a fast ($O(r^3 n \log(n))$) and stable algorithm to transform this hierarchical representation into a so-called unitary-weight representation, as introduced in an earlier work of the authors. This reduction allows the use of fast and stable unitary-weight routines (or by the same means, fast and stable routines for sequentially semiseparable (SSS) and quasiseparable representations used by other authors in the literature), leading, e.g, to direct methods for linear system solution and for the computation of all the eigenvalues of the given hierarchically rank structured matrix.

Key Words.
hierarchically semiseparable (HSS) matrix, $\mathcal{H}^2$-matrix, low rank submatrix, tree, QR factorization, unitary-weight representation

AMS Subject Classifications.
65F30, 15A03

189 Error analysis and computational aspects of SR factorization via optimal symplectic Householder transformations. *A. Salam and E. Al-Aidarous*.

Abstract.
Symplectic QR like methods for solving some structured eigenvalue problems involves SR factorization as a key step. The optimal symplectic Householder SR factorization (SROSH algorithm) is a suitable choice for performing such a factorization. In this paper, we carry out a detailed error analysis of the SROSH algorithm. In particular, backward and forward error results are derived. Also, the computational aspects of the algorithm (such as storage, complexity, implementation, factored form, block representation) are described. Some numerical experiments are presented.

Key Words.
skew-symmetric inner product, optimal symplectic Householder transformations, SR factorization, error analysis, backward and forward errors, implementation, factored form, WY factorization, complexity

AMS Subject Classifications.
65F15, 65F50

207 Algebraic properties of the block GMRES and block Arnoldi methods. *L. Elbouyahyaoui, A. Messaoudi, and H. Sadok.*

Abstract.
The solution of linear systems of equations with several right-hand sides is considered. Approximate solutions are conveniently computed by block GMRES methods. We describe and study three variants of block GMRES. These methods are based on three implementations of the block Arnoldi method, which differ in their choice of inner product.

Key Words.
block method, GMRES method, Arnoldi method, matrix polynomial, multiple right-hand sides, block Krylov subspace, Schur complement, characteristic polynomial

AMS Subject Classifications.
65F10