

ON MODIFIED ASYMPTOTIC SERIES INVOLVING CONFLUENT HYPERGEOMETRIC FUNCTIONS*

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Abstract. A modification of the Poincaré-type asymptotic expansion for functions defined by Laplace transforms is analyzed. This modification is based on an alternative power series expansion of the integrand, and the convergence properties are seen to be superior to those of the original asymptotic series. The resulting modified asymptotic expansion involves a series of confluent hypergeometric functions $U(a, c, z)$, which can be computed by means of continued fractions in a backward recursion scheme. Numerical examples are included, such as the incomplete gamma function $\Gamma(a, z)$ and the modified Bessel function $K_\nu(z)$ for large values of z . It is observed that the same procedure can be applied to uniform asymptotic expansions when extra parameters become large as well.

Key words. confluent hypergeometric functions, asymptotic expansions, saddle point method, convergence and divergence of series and sequences

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